

# Microwave diagnostics of magnetoactive plasmas

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(1959 – 2009, 50-th anniversary)

(1977-84: tokamak TM-1-MH  
1984-2007: tokamak CASTOR  
2008->: tokamak COMPASS)

## Outline:

- propagation of electromagnetic waves through a magnetoactive plasma (waves change phase, amplitude, polarization and even frequency)
- I. active methods based mostly on dependence of the **cold** plasma **refractivity index** of the electron density (exception point 3)
  - 1) direct **measurement of electron density** (including its space profile)
    - **microwave interferometry** measuring the phase of the **fix (or nearly fix) frequency** probing **O wave**, passing the whole plasma perpendicularly to the direction of the confining magnetic field (homodyne or heterodyne systems, one or multichannel systems, moreover with possibility of polarimetric determination of the current density  $j(r)$  profile) → **line integrated** method
    - **microwave reflectometry** with fast **frequency sweeping** of the probing wave (change of location of the wave reflection with the frequency) → edge, **local**
  - 2) observation of local **density turbulent characteristics and directed flows** inside the plasma due to the **reflection/scattering** of **constant frequency**, well focused beam of probing wave (the spot of the beam determines the method space resolution), due to existence of the space localized inhomogeneities (study of anomalous transport and L→H transition, ELM's etc.)
    - **scattering** (i.e.  $f \rightarrow f + \delta f$ ) of the probing beam in **different observation direction** →  $\omega$ - as well as  $k$ -spectra of these turbulent formations can be obtained
    - **Doppler reflectometry** (the beam is launched and also detected under not zero angle to the normal of the moving reflecting surface, i.e. **scattering order  $m=-1$**  is observed) → → plasma rotation
  - 3) **ECA - Electron Cyclotron resonance wave Absorption** measurements (based on thermal effect at the wave's resonance)
- II. passive methods of **electron temperature profile** measurement by absolute calibrated **ECE radiometer**, based on electron **thermal effect** at **ECR** (see ECA as well) (in additional to the thermal radiation suprathreshold radiation of group of non maxwellian particles and EBW mode conversion can be also observable); simple "sniffers" (correlation characteristics of the heating RF wave)

# Microwaves - frequencies and generators used

f [Hz]	$\lambda$ [mm]	
$10^9$	300	klystrons, BWO, TWO, Gunn oscillators, avalanche diodes
$10^{10}$	30	- ,, -
$10^{11}$	3	<b>mm waves</b> (doublers, triplers, ...) → <b>submm waves</b> → → <b>FIR lasers</b>
	1,22	$C^{13}H_3F$ isotropic methyl fluoride
	0,432	HCOOH formic acid (RTP – 2x40mW, optically pumped)
	0,381	DCOOD deuterized formic acid
$8,9 \cdot 10^{11}$	0,337	HCN cyanide hydrogen (electrically pumped)
$> 10^{12}$	0,119	DCN
	0,118	CH <sub>3</sub> OH methyl alcohol

**increasing frequencies** -> transition from basic to the oversized waveguides and finally to the quasioptics and open resonators (lasers)

# Propagation of the waves through a medium

(plasma dispersion equation → effect of the plasma parameters)

- fields and medium properties are coupled (through Maxwell equations) by two **quantities**  $\epsilon$  and  $\mu$  (permittivity,  $D=\epsilon E$  and permeability,  $B=\mu H$ )
  - isotropic medium  $\Rightarrow \epsilon, \mu$  are scalars
  - anisotropic medium  $\Rightarrow \epsilon, \mu$  are generally complex tensors of the 3<sup>rd</sup> order
- relation between **local values of  $\epsilon, \mu$**  characterizing medium as a whole and **local microscopic parameters of the plasma** (distribution function and density of all kinds of particles) is derived by plasma **kinetic theory**
- plasma in magnetic field  $\Rightarrow \epsilon = \epsilon_{rel} \cdot \epsilon_0$  is a complex tensor ( $\epsilon_0 = 8,85 \cdot 10^{-12} \text{F/m}$ ), while  $\mu \doteq \mu_0 = 4\pi \cdot 10^{-7} \text{H/m}$  is practically constant
- using **Maxwell equations** and supposing a **harmonic plane** wave ( $\sim e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})}$ ), the following **wave equation** (i.e. possible time and space oscillating electromagnetic fields) can be easily derived (for space discharge  $\rho=0$ ):
$$\Delta \mathbf{E} + (\omega/c)^2 \epsilon_{rel} \mathbf{E} = 0 \quad (\text{knowing that } \epsilon_0 \mu_0 = 1/c^2)$$
- if we denote  $z$  as direction of the wave propagation then solution of this equation can be written as follows:
$$\mathbf{E} = \mathbf{E}_0 \cdot \exp[-i(\omega t - \mathbf{k} \cdot \mathbf{z}) - \alpha z] \quad \rightarrow v_{ph} = dz/dt = \omega/k \quad (=c/n, n < 1 \rightarrow v_{ph} > c)$$
- where  $\text{Re}k = k = (\omega/c) \cdot n = 2\pi/\lambda$  denotes the **wave number** and  $n$  **index of refractivity**, determining the **phase velocity** and in an inhomogeneous plasma also the wave trajectory – Ray tracing (note that generally  $n$ , as well as  $k$ , are vectors, which components and therefore also the amplitudes depend on direction of the wave propagation with regards to the density gradient and if magnetic field is present, also to direction of the magnetic field !!!)
- and  $\text{Im}k = \alpha$  is **absorption coefficient** (determining the wave attenuation)

- relation between the index of refractivity of the wave with a given frequency and microscopic parameters of the plasma is given by **dispersion equation**
- **dispersion equation** can be derived as condition for a **non-trivial solution** of three equations for all three components of the wave equation for the plane wave electric field  $E \sim e^{i\mathbf{k} \cdot \mathbf{r}}$  (determinant of the equations coefficients equals zero), overwritten in the vector form:

$$\mathbf{n} \times \mathbf{n} \times \mathbf{E} + \epsilon_{\text{rel}}(\omega, \mathbf{k}) \cdot \mathbf{E} = 0 \quad (\text{again, } \mathbf{n} = (c/\omega) \cdot \mathbf{k} \text{ is a vector})$$

- if we put direction of the magnetic field **B in z axis** and direction of the wave propagation **k in the plane z,x**, then vector  $\mathbf{n}$  has only two components (parallel  $n_{\parallel}$  and perpendicular  $n_{\perp}$  to  $\mathbf{B}$ )
- in such case the condition of the non-trivial  $\mathbf{E}$  solution can be written as an algebraic equation of the sixth power for the vector  $\mathbf{n}$  amplitude (the third power for  **$n^2 = n_{\parallel}^2 + n_{\perp}^2$** ):

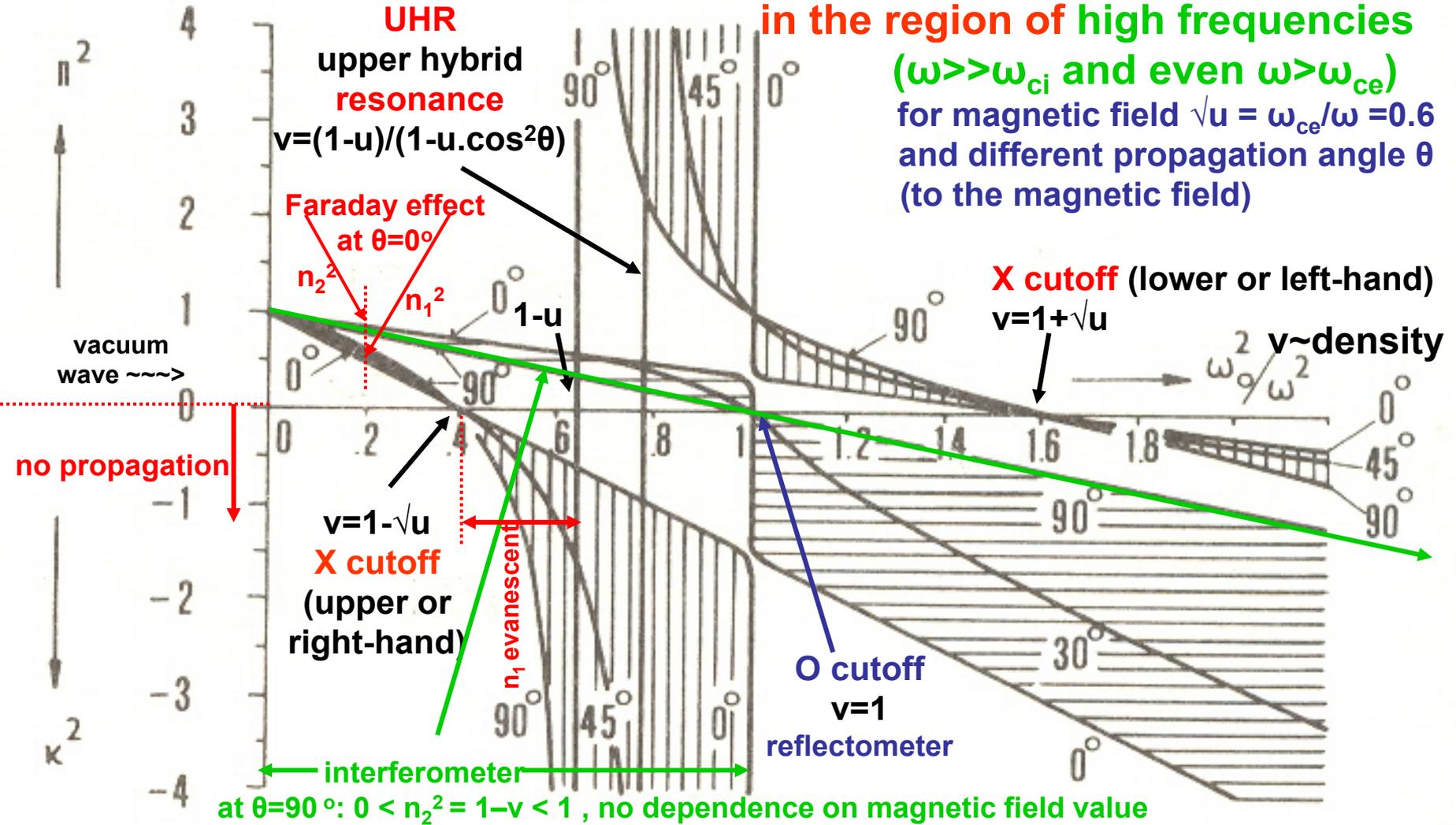
$$\cancel{n^6} \cdot A + n^4 \cdot B + n^2 \cdot C + D = 0,$$

where all coefficients are known function of components of the tensor  $\epsilon_{\text{rel}}$  and of the wave propagation direction, whereas coefficient  **$A \sim v_T^2/c^2$** , therefore in the **cold plasma  $A \rightarrow 0$**

- it may be seen that in the **cold plasma dispersion equation** becomes to be quadratic equation for  $n^2$ ; its **two solutions  $n_1^2 < n_2^2$**  ( $n_1$  fast,  $n_2$  slow) are two different branches of the transverse electromagnetic waves (with different polarization), which can propagate through the cold magnetoactive plasma
- in the case of a **cold plasma**, solution of dispersion equation are entering plasma **density**, **collision** frequency and cyclotron frequency of all charged particles (i.e. value of the **magnetic field**), further **direction** of the wave propagation;
- nearly all **microwave diagnostics** are based on the  $n_2$  **cold collisionless plasma approximation**; attention in the case of **resonances**, where  **$n \rightarrow \infty$  !!!** and the **thermal velocity** of particles must be added  **$\rightarrow T_e$  from ECE or ECA**

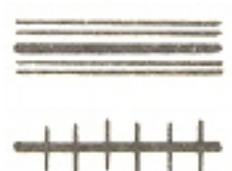
# Dispersion equation of a cold collisionless plasma in the region of high frequencies

( $\omega \gg \omega_{ci}$  and even  $\omega > \omega_{ce}$ )  
for magnetic field  $\sqrt{u} = \omega_{ce}/\omega = 0.6$   
and different propagation angle  $\theta$   
(to the magnetic field)



$(\omega_o/\omega)^2 = v = N/N_{crit} \sim \text{density}$   
here  $\omega_o = \text{plasma frequency}$   
 $(\omega_{ce}/\omega)^2 = u \sim \text{mag. field}$

**Terminology:**  
slow wave  $n_2$   
( $n_2 > n_1 \rightarrow v_2^{ph} < v_1^{ph}$ )  
fast wave  $n_1$



$E \parallel B$   
ordinary wave

LHCPW

$E \perp B$   
extraordinary wave

RHCPW

$\theta = 90^\circ$

$\theta = 0^\circ$

propagation angle

arbitrary

# Cold plasma wave classification (Allis diagram)

ICR ( $\omega_{pe} \gg \omega_{pi} \sim \omega$ )

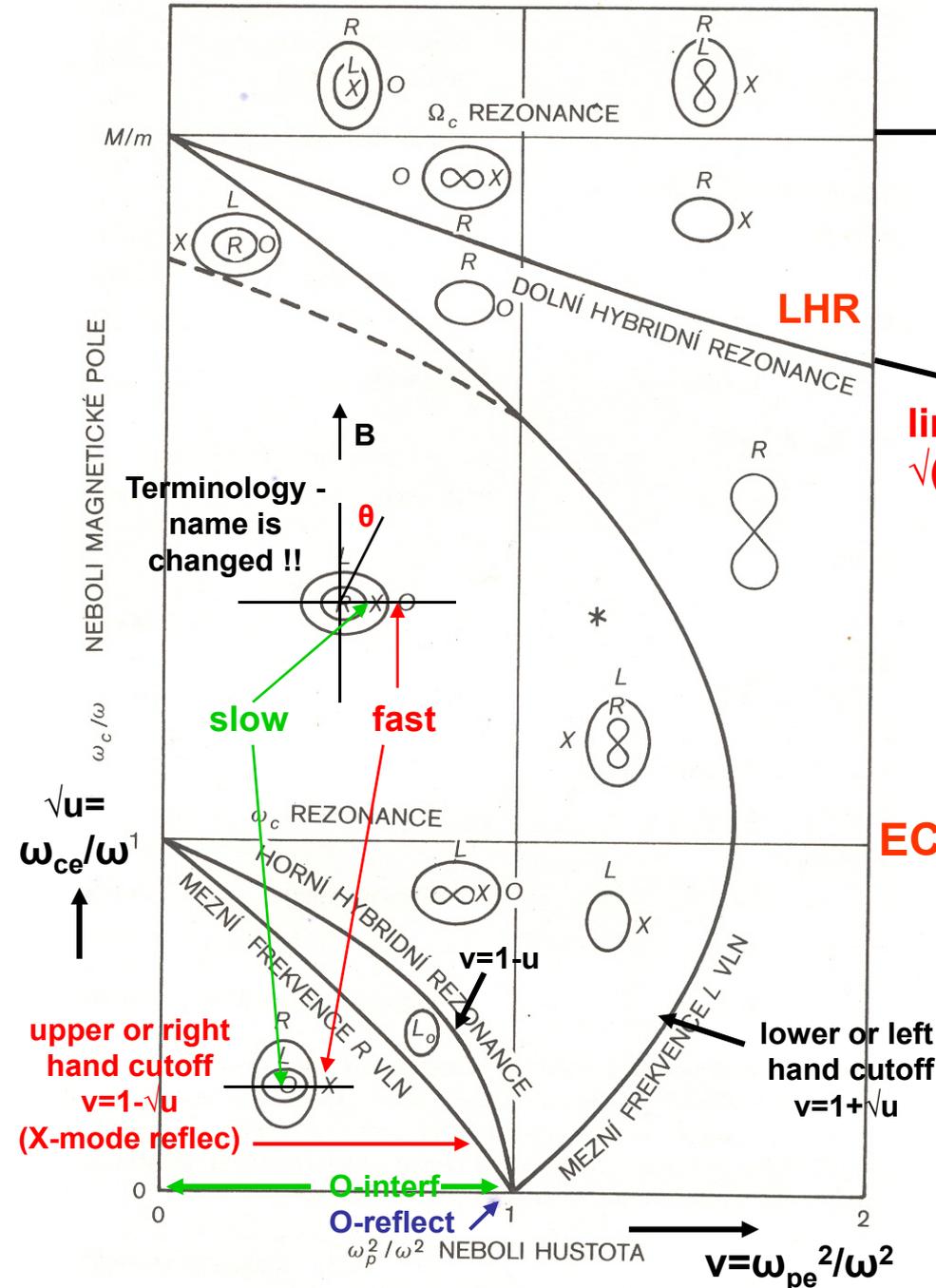


Diagram of normalized magnetic field versus normalized density (normalized to the wave frequency)

- fragmented into a set of regions, mutually separated by cut-offs (limit frequencies) and wave resonances

Closed curved (one, two or also no) inside the regions show, which waves (fast and slow) can propagate in this region and simultaneously give, in polar coordinates, dependence of the phase velocity of these waves (as curve radius) on the angle  $\theta$  of the wave propagation with regards to the magnetic field (zero angle must be put in the vertical axis)

Remark.: diagram is sometimes called also CMA diagram (Clemmow-Mullaly-Allis)



# Refractivity index frequency dependence of O-wave - comparison of the effect of all other particles presented

- there are *three groups of particles*, which influence the wave refractivity index:  
**free electrons**, **free ions** and **neutrals** (through the bounded electrons)
- the contributions of all these groups to the total refractivity index can be written symbolically in cumulative form as follows:  
$$n = 1 + \delta n_e + \delta n_i + \delta n_n$$
- we will shown further that for the *cold collisionless* plasma with  $N_e \ll N_{crit}(\lambda)$   
$$\delta n_e = -4,48 \cdot 10^{-14} \lambda^2 N_e < 0 \quad [\text{cm}, \text{cm}^{-3}]$$
  
(quite different situation arises in the case of strong collisions !!! - see the next slide)
- it is clear that  $\delta n_i = (Z^2 \cdot m) / M \cdot \delta n_e \leq 5 \cdot 10^{-4} \cdot \delta n_e$  (for H), i.e.  $\delta n_i \ll \delta n_e$  (both are negative)  
→ effect of ions can be neglected
- contribution of the neutral particles is given by ability of these particles to be polarized; this contribution has two parts
  - the first is frequency dependent, the second doesn't depend on the frequency:  
$$\delta n_n = \alpha(1 + \beta/\lambda^2) \cdot N_n > 0 \quad (\text{positive})$$
  
here  $\alpha$  and  $\beta$  are tabulated constants of single gases;  
 $\delta n_n$  is often also expressed, using only one constant K (so called Gladstone-Daley constant, however depending on frequency) in the form  $\delta n_n = K \cdot \rho$   
(here  $\rho$  [g/cm<sup>3</sup>] is the gas mass density)
  - contribution of the neutral particles has the **opposite sign** than contribution of those charged particles !!!

## Note the important fact:

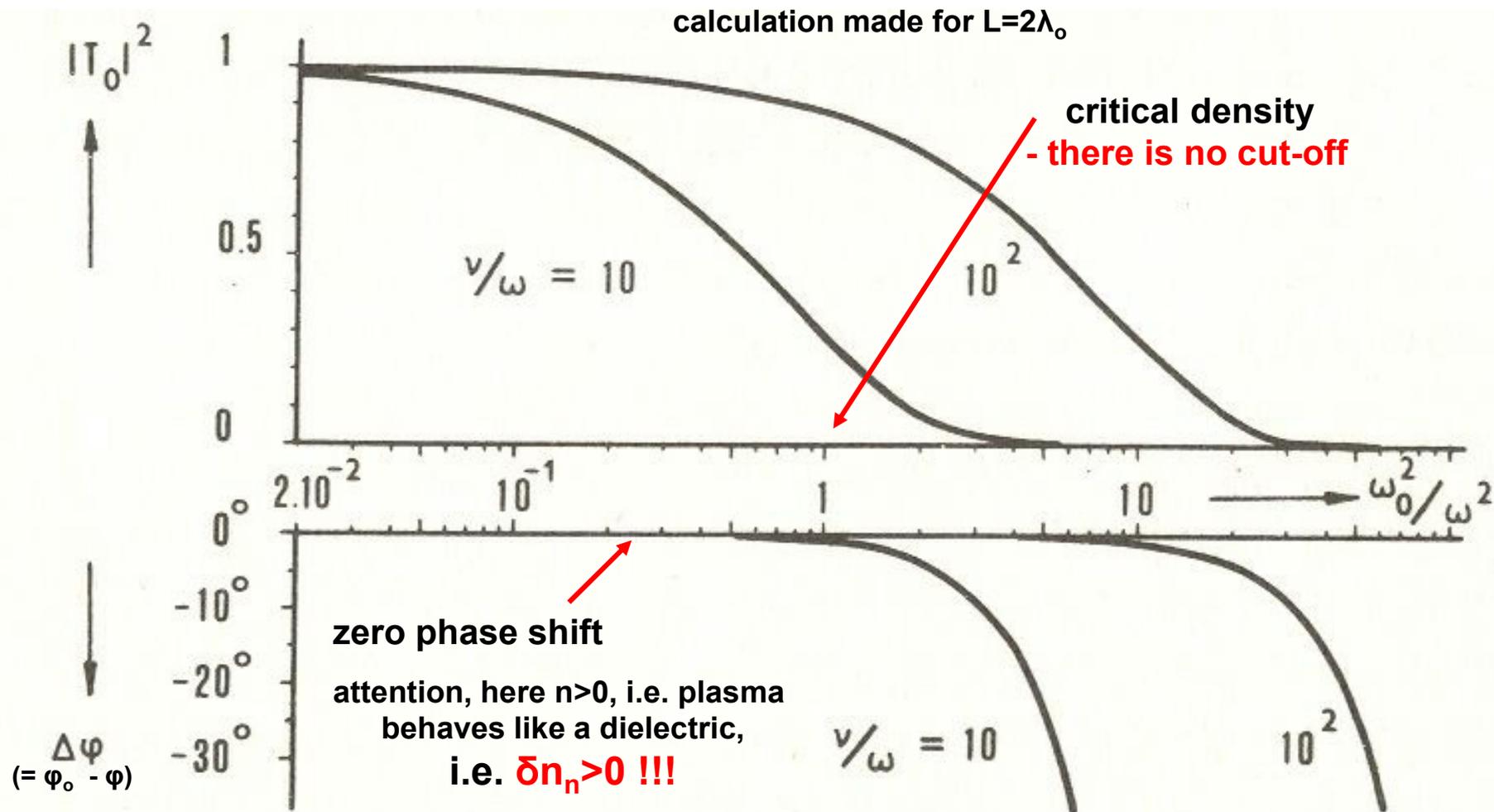
Contributions of the charged and the neutral particles to the index of refractivity exhibit (in the *collisionless* plasma) quite **opposite frequency dependence** ( $\delta n_e$  decreases, while  $\delta n_n$  increases at transition from microwaves to FIR and farther to the IR and visible light)

→  $n < 1$  as well as  $n > 1$  is possible to measured in low ionized gases at highest frequencies !!!

# Dependence of the power transmission coefficient and phase shift of the ordinary wave having frequency $\omega$ on the normalized electron density $N/N_{\text{crit}}$ of the high collisional plasma

(high pressure discharges)

(Musil, Žáček: Z. f. Naturforsch. 30a (1975) 947)



# Phase shift of the O-mode wave when the plasma layer L is created

- phase of the wave with frequency  $\omega$  and vacuum wavelength  $\lambda_v$  after passing the distance L in vacuum:  $\varphi_v = 2.\pi.(L/\lambda_v)$
- phase of the same wave after passing of the same distance L in a medium with **index of refractivity  $n(x)$**  changing along the wave trajectory x:

$$\varphi = 2.\pi.(1/\lambda_v). \int_0^L n(x).dx$$

- for **ordinary** wave is valid:  $n(x) = (\epsilon_{rel})^{1/2} = (1 - N/N_{crit})^{1/2}$  [ $\epsilon_{rel} = 1 - (\omega_o/\omega)^2$ ,
- if  $N \ll N_{crit}$  is valid, then:  $n(x) = 1 - (1/2).(N/N_{crit})$  where  $\omega_o^2 = N.e^2/m\epsilon_o$ ]
- if plasma is created, we can write the phase difference (shift) as following

$$\Delta\varphi = \varphi - \varphi_v = 2.\pi.(1/\lambda_v). \left( L - (1/2). \int_0^L (N/N_{crit})dx - L \right)$$

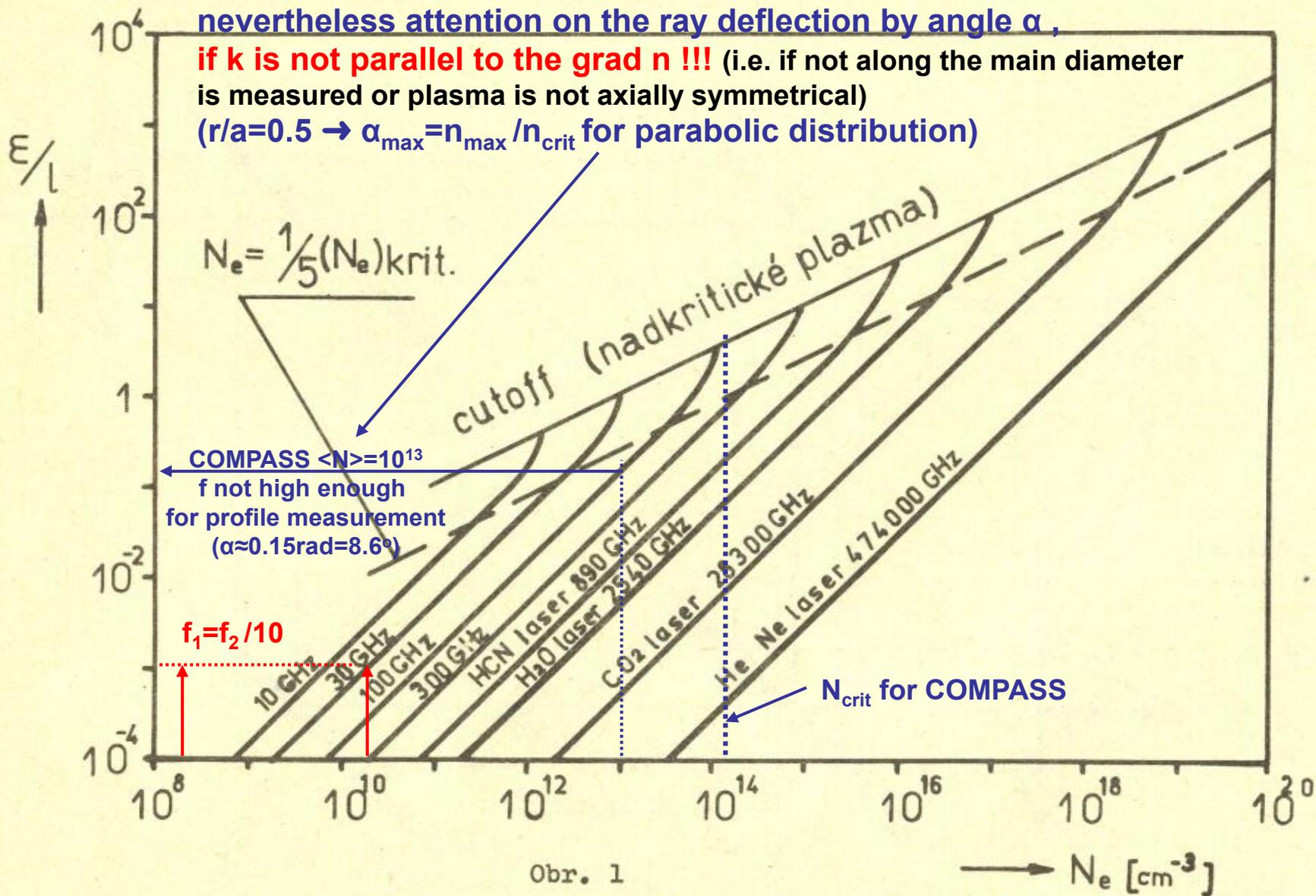
- after introduction of the line averaged value of **density N**

$$\langle N \rangle = (1/L). \int_0^L N(x).dx \quad \rightarrow \quad \Delta\varphi = -\pi.(L/\lambda_v).(\langle N \rangle/N_{crit})$$

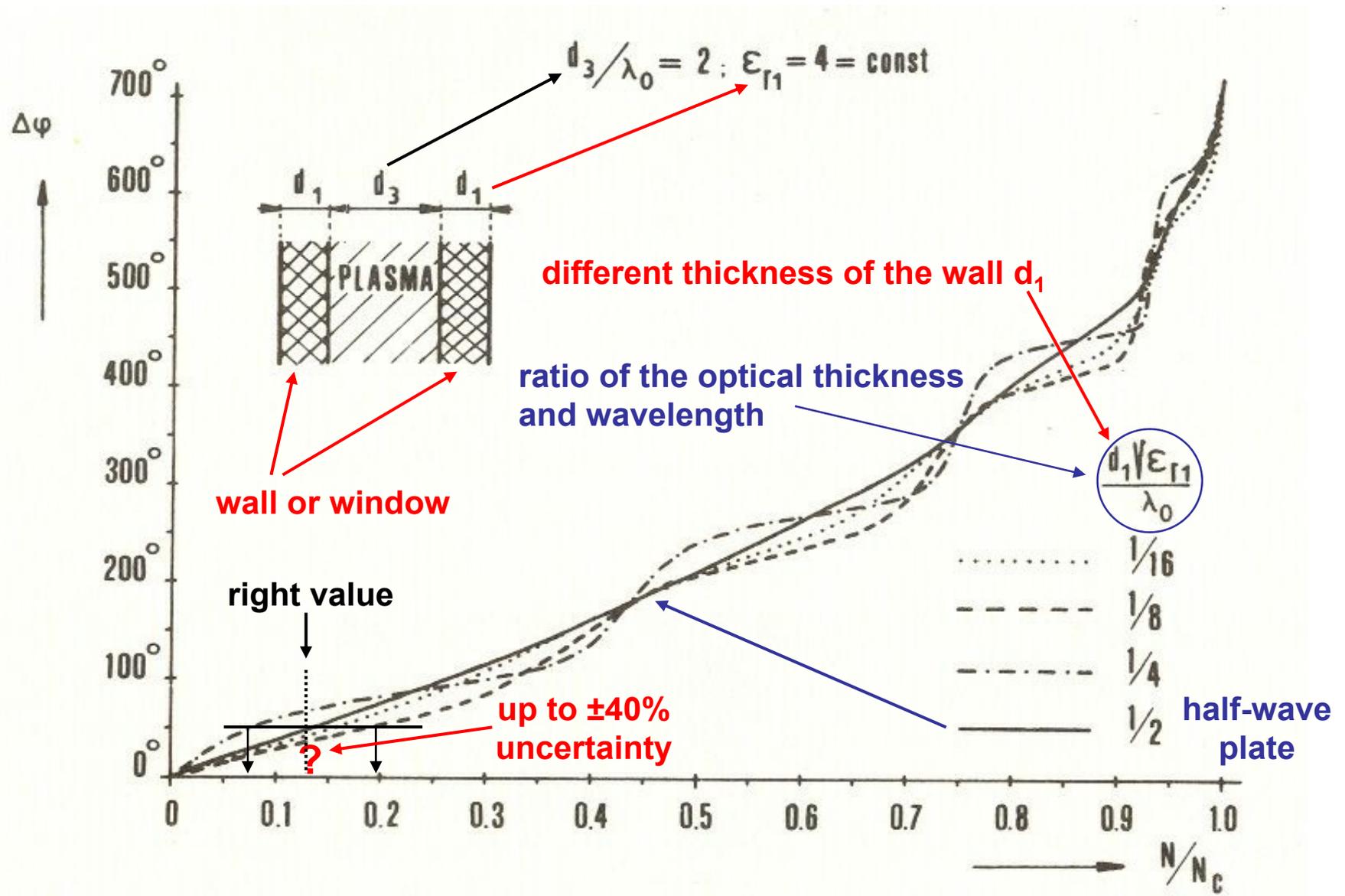
COMPASS  $\sim 500/2.3=220, 2.10^{20}m^{-3}$

**Conclusion:** the phase shift measured by interferometer increases linearly with  $\lambda_v$  of the probing wave used (doesn't increase with the second power  $\lambda_v$  as the  $N_{crit}$ , due to decreasing relative path  $L/\lambda_v$  of the wave due to the  $L=const$ )

# Dependence of the relative phase shift $\epsilon = \Delta\phi/2\pi$ , related to 1cm trajectory, on the electron density for different frequency of the probing wave



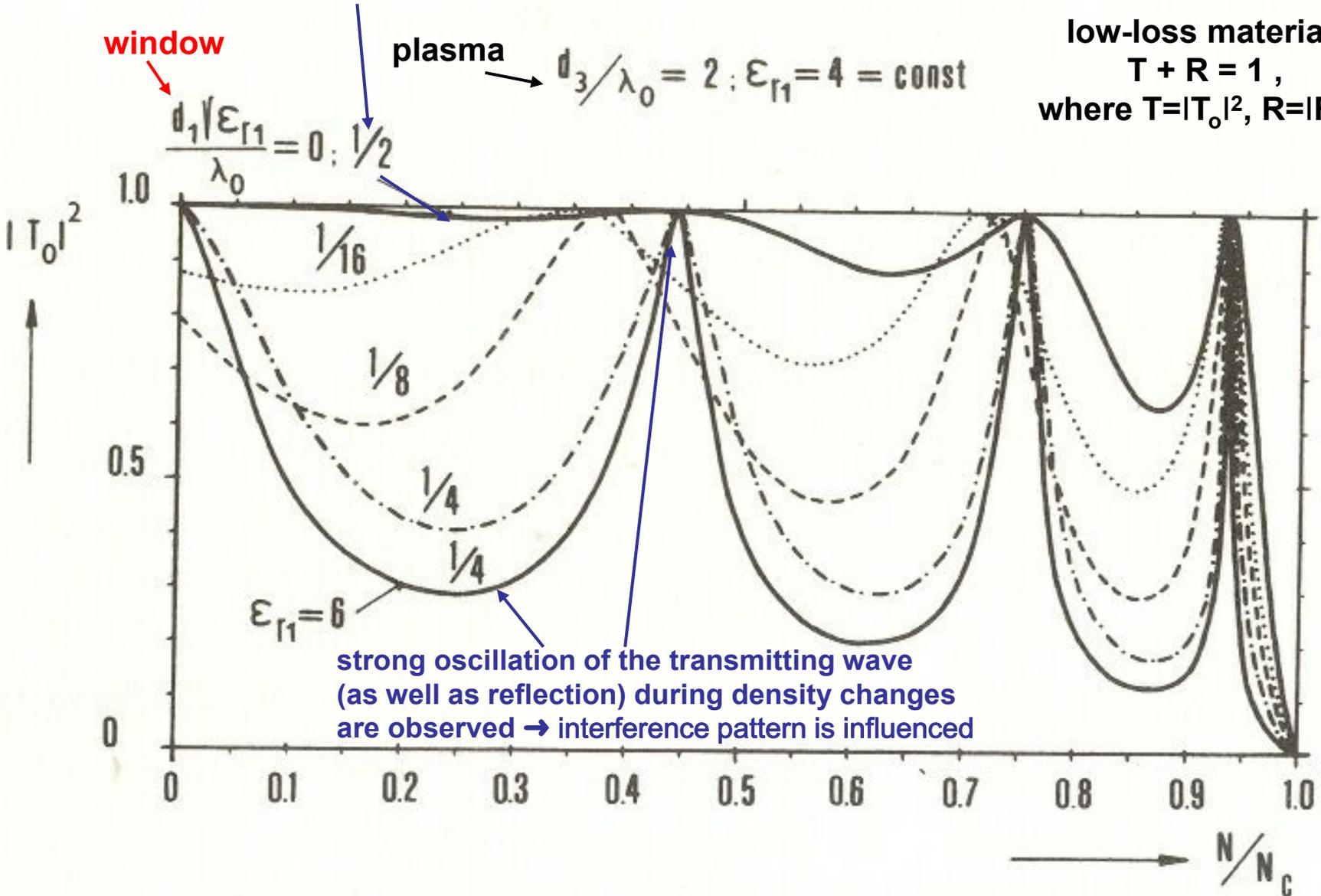
# Distortion of the probing wave phase by two dielectric walls surrounding the plasma



# Density dependence of the wave power transmission coefficient if the plasma is surrounded by two dielectric walls

again see the half-wave plate (important for RF generators)

low-loss material:  
 $T + R = 1$ ,  
 where  $T = |T_o|^2$ ,  $R = |R_o|^2$



$d_1/\lambda_0 = 0; 1/2$

$d_3/\lambda_0 = 2; \epsilon_{r1} = 4 = \text{const}$

$\epsilon_{r1} = 6$

strong oscillation of the transmitting wave  
 (as well as reflection) during density changes  
 are observed → interference pattern is influenced



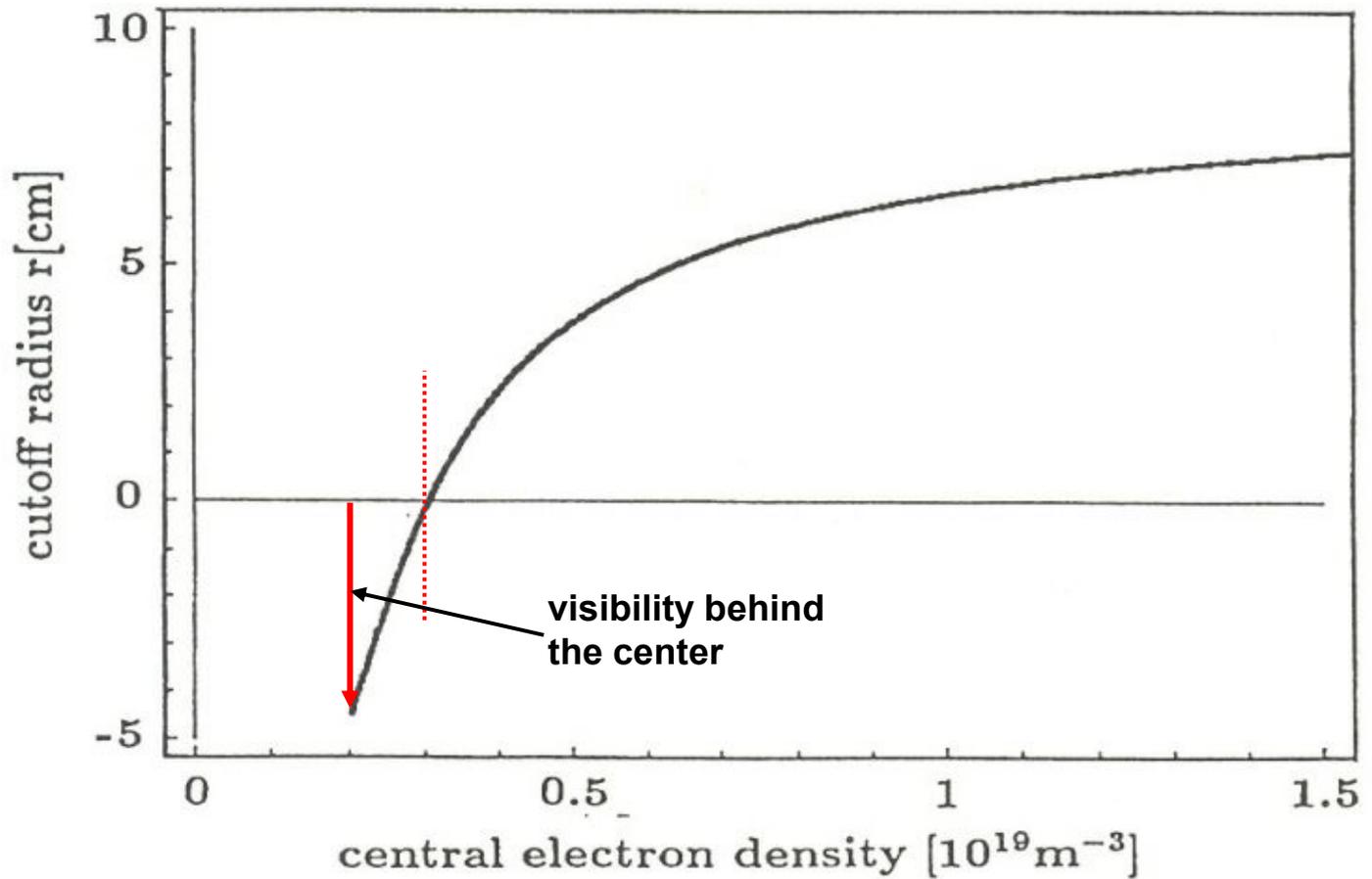
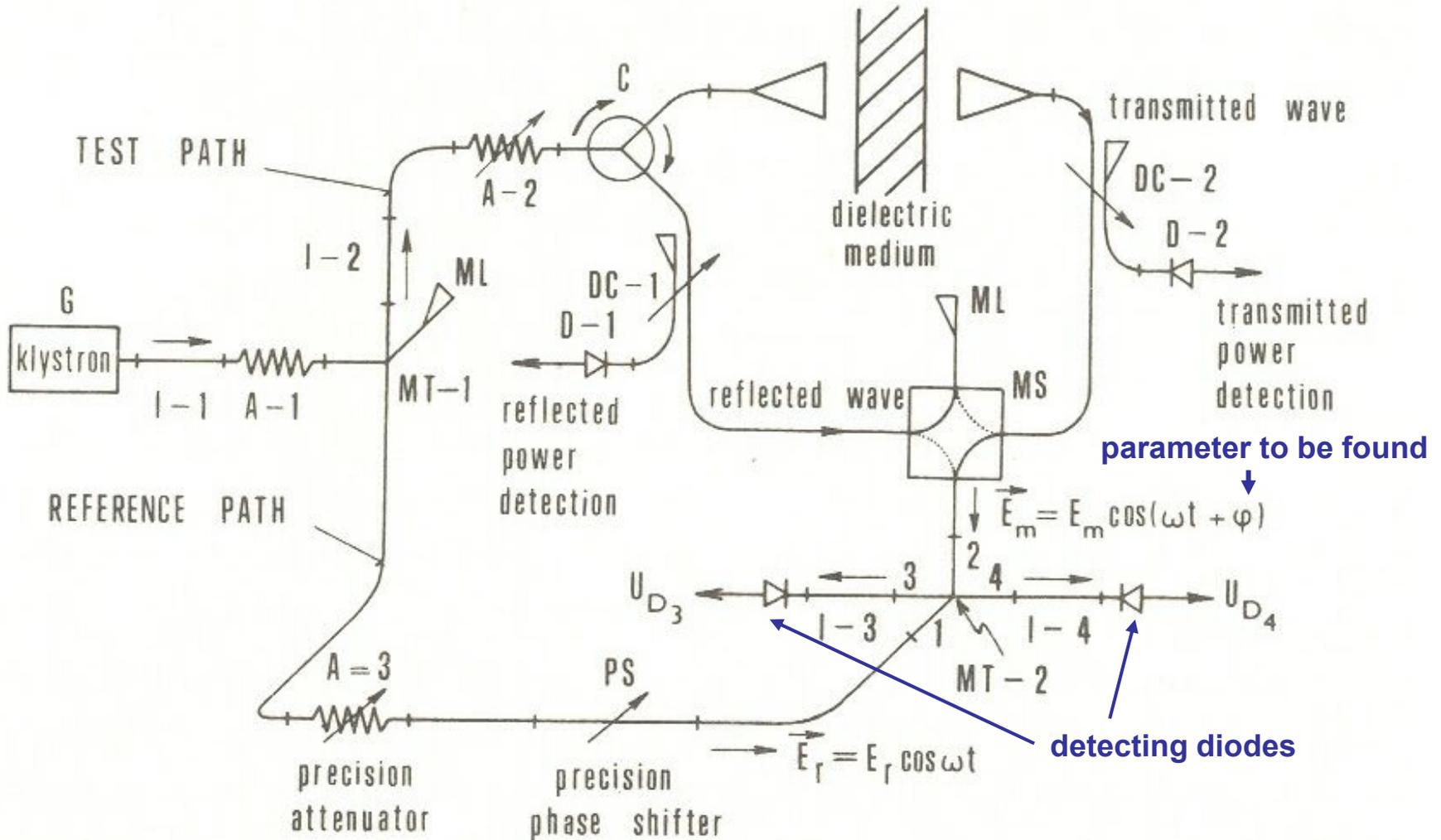


Figure 2: Dependence of the cut-off radius  $r_c$  on the central plasma density in CASTOR.

(Using the X-mode reflectometer working on the frequency 35GHz, if  $B(0)=1\text{T}$  )

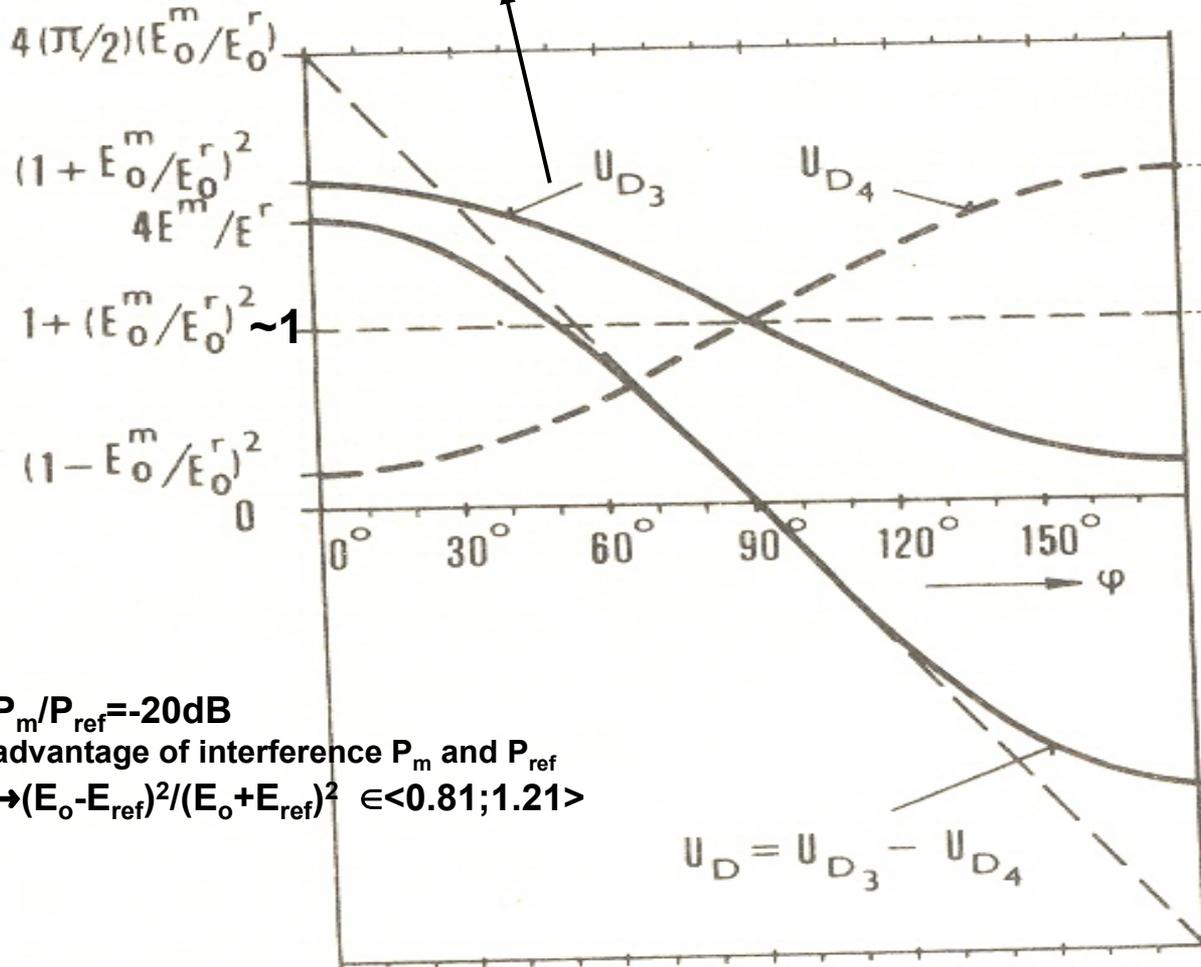
# Scheme of the microwave homodyne interferometer for measurement of the transmitted as well as reflected waves complex amplitude



# Dependence of normalized voltage $U_{D3}$ and $U_{D4}$ of detection diodes D3 and D4 of the mixing Magic-T junction on the phase difference $\varphi$ between the reference $E_o^{ref} = \text{const}$ and measuring $E_o^m(t, \varphi)$ waves

diodes are quadratic  $\rightarrow U_3 \sim E_3^2 = (E_o^{ref})^2 / 2 \cdot [1 + (E_o^m / E_o^{ref})^2 + 2(E_o^m / E_o^{ref}) \cdot \cos\varphi]$

usually  $E_o^m < E_o^r$ ,  
therefore  $(E_o^m / E_o^r)^2 \ll 1$



### Conclusions:

- 1) Amplitude of the interference signal is function not only of the waves **phase difference**, but also of the **wave amplitude** (which is changing in the time)
- 2) Only passages through center of the interferometric picture ( $\varphi = \pi/2, 3\pi/2, \dots$ ) are on the measuring wave amplitude independent.

$P_m / P_{ref} = -20\text{dB}$

advantage of interference  $P_m$  and  $P_{ref}$

$\rightarrow (E_o - E_{ref})^2 / (E_o + E_{ref})^2 \in \langle 0.81; 1.21 \rangle$

# 4-beam interferometer with double elliptical cylinder mirrors

(8mm, Musil, Žáček: Czech.J. Phys.B 16 (1966) 782)

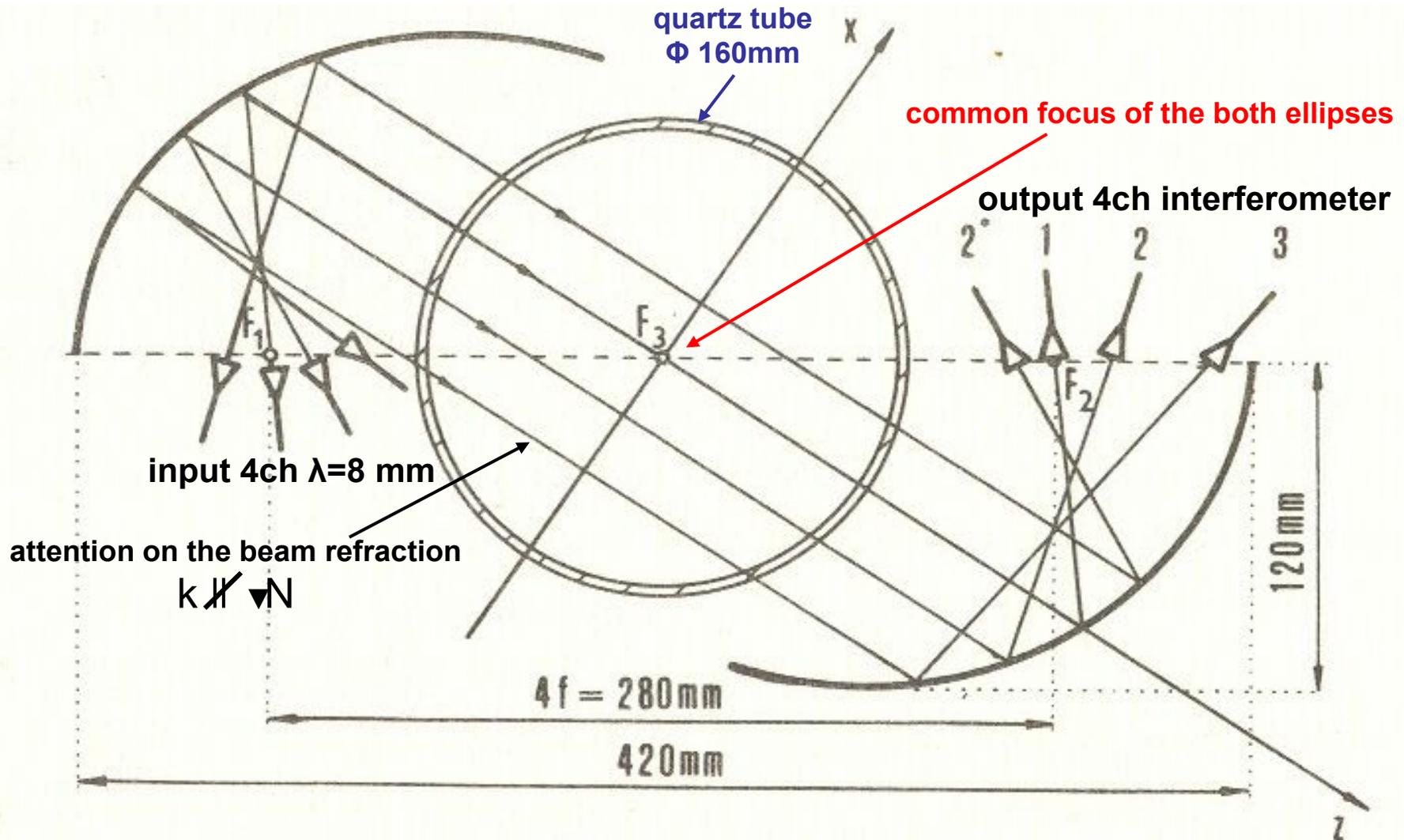
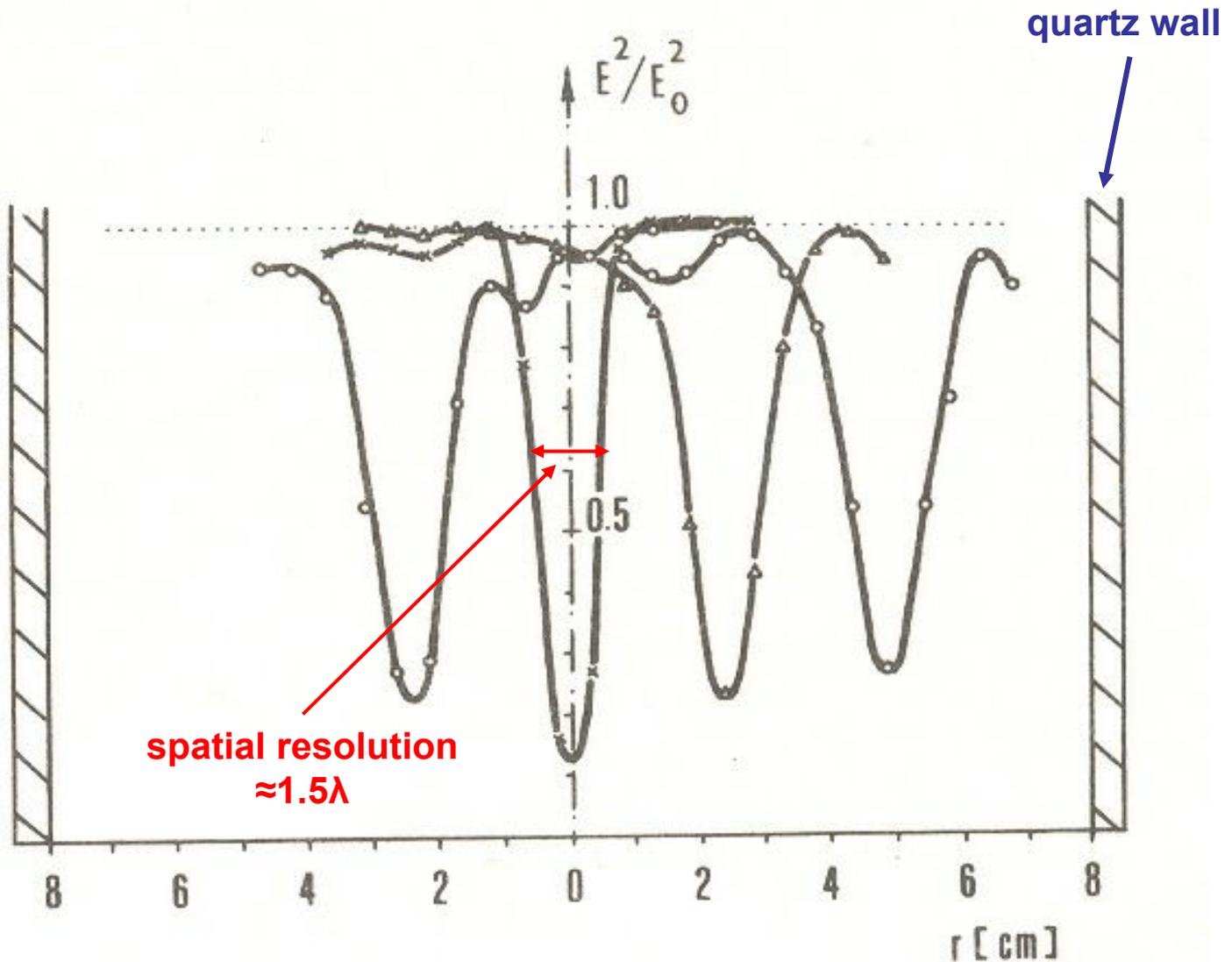
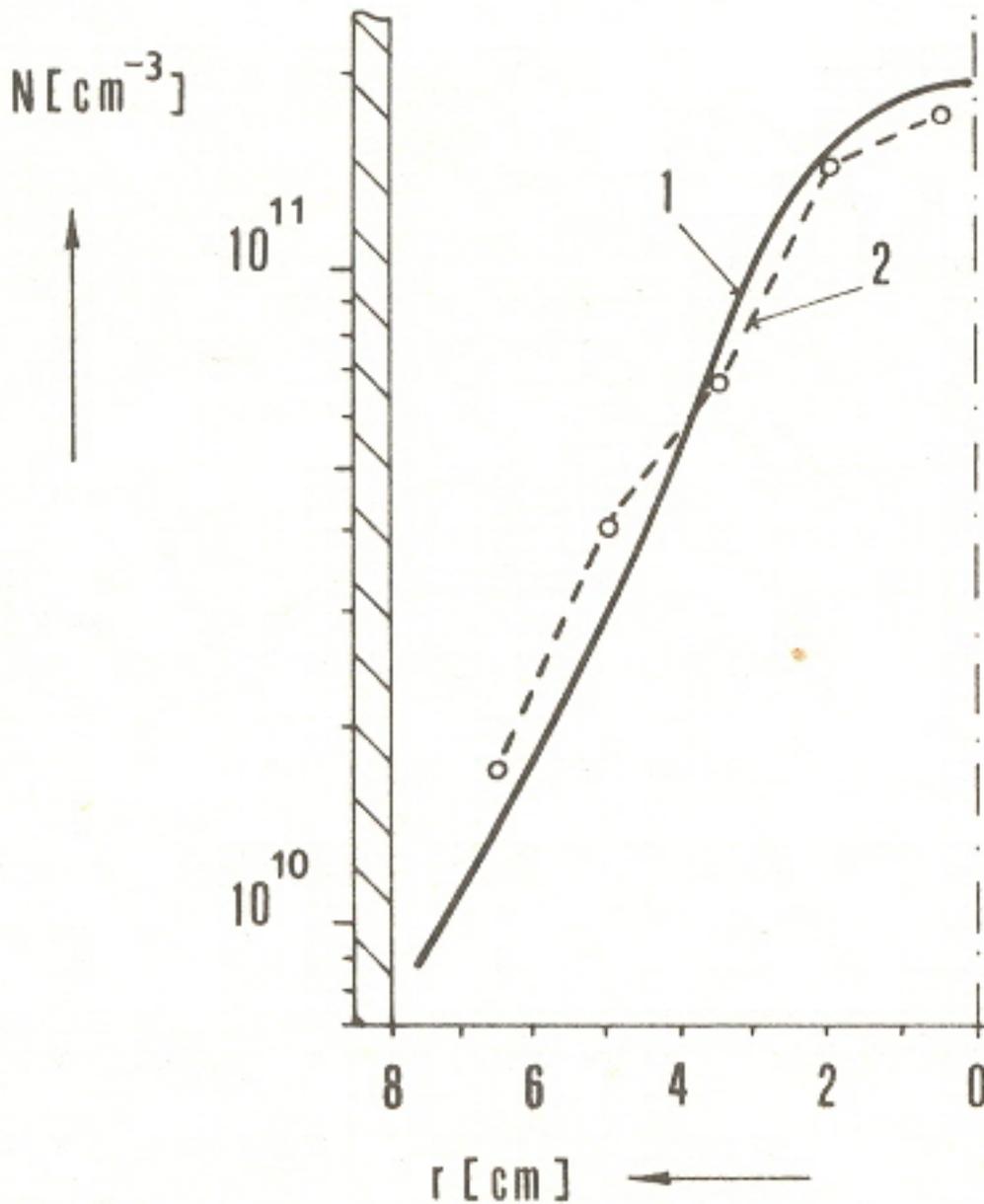


Fig. 5.32 Antenna system of a multiple-beam interferometer.

# Radial power profiles of 4 beams of 8mm interferometer, formed by double elliptical mirrors

- no overlapping of the channels



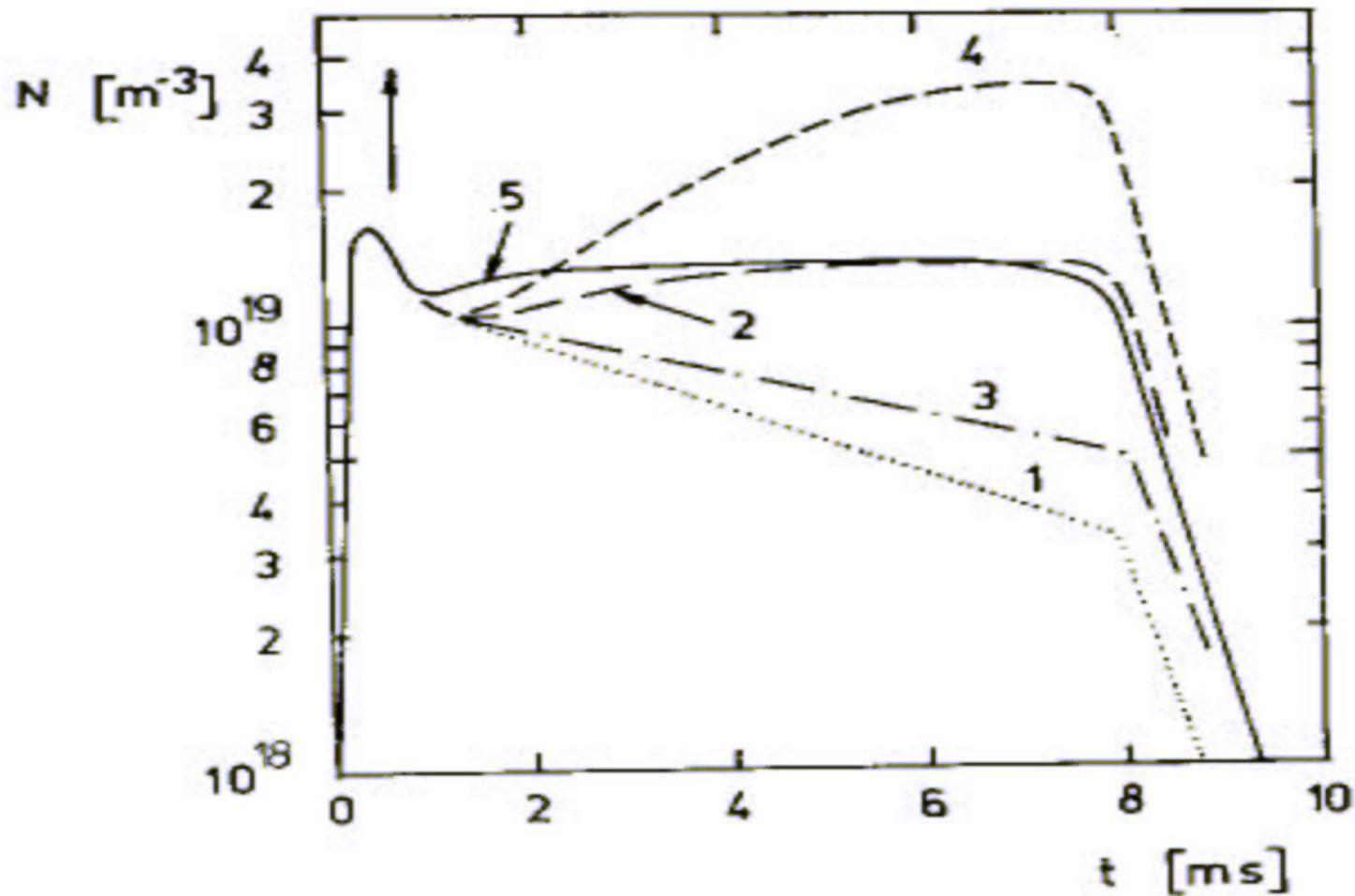


**Comparison of the radial profile of the electron density in a cylindrical argon reflex discharge plasma**

full line 1 – microwaves  
(fit from the 4 channels)  
dashed line 2 – Langmuir probes

**Stationary discharge with following parameters:**  
current 2A  
mag. field 0,05T  
pressure  $1,3 \cdot 10^{-3}$ Pa

**Line average density in TM-1-MH with a different impulse gas puffing**  
(evaluated still by laborious method by the zero crossing points of the interferogramme)

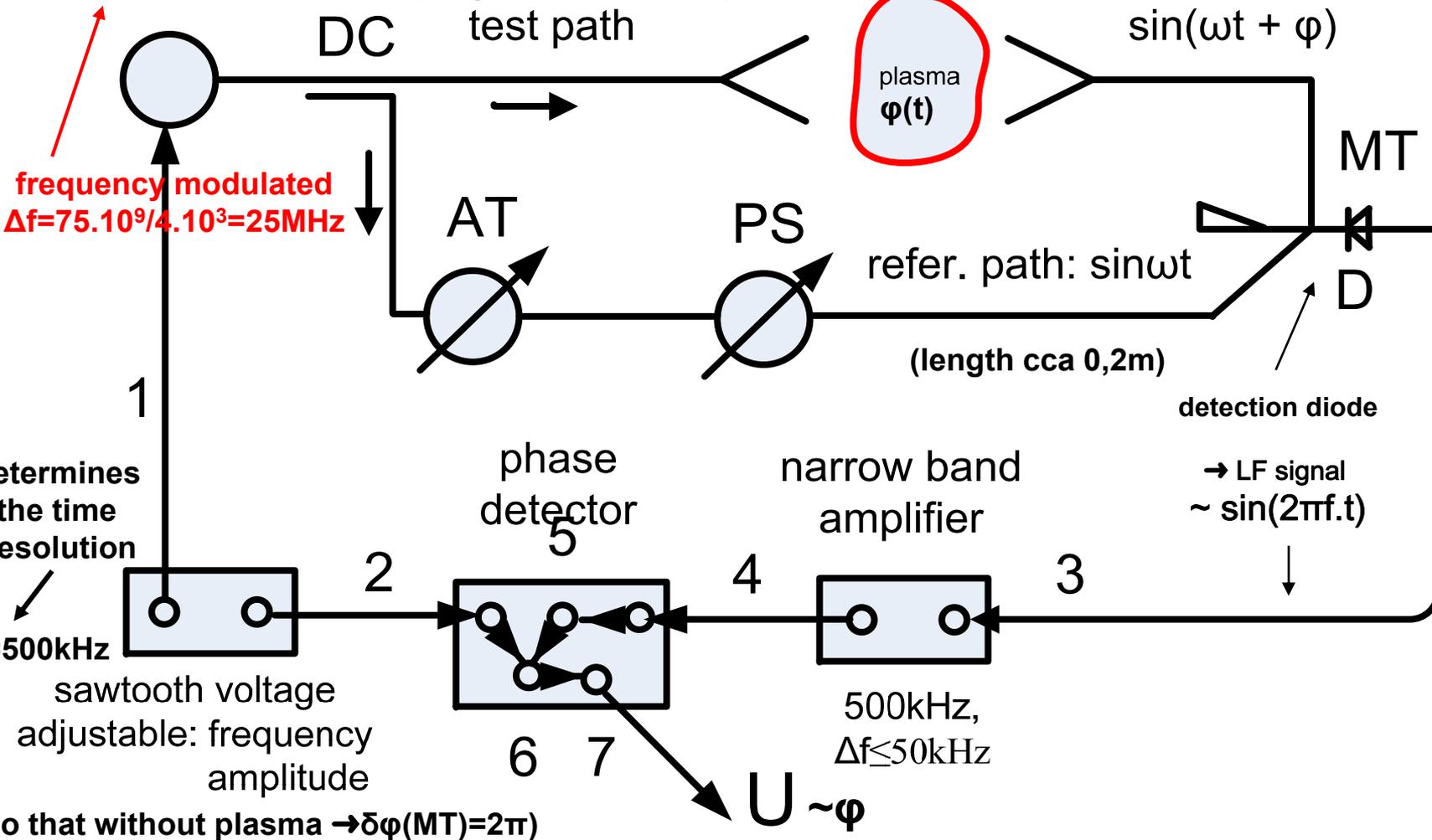


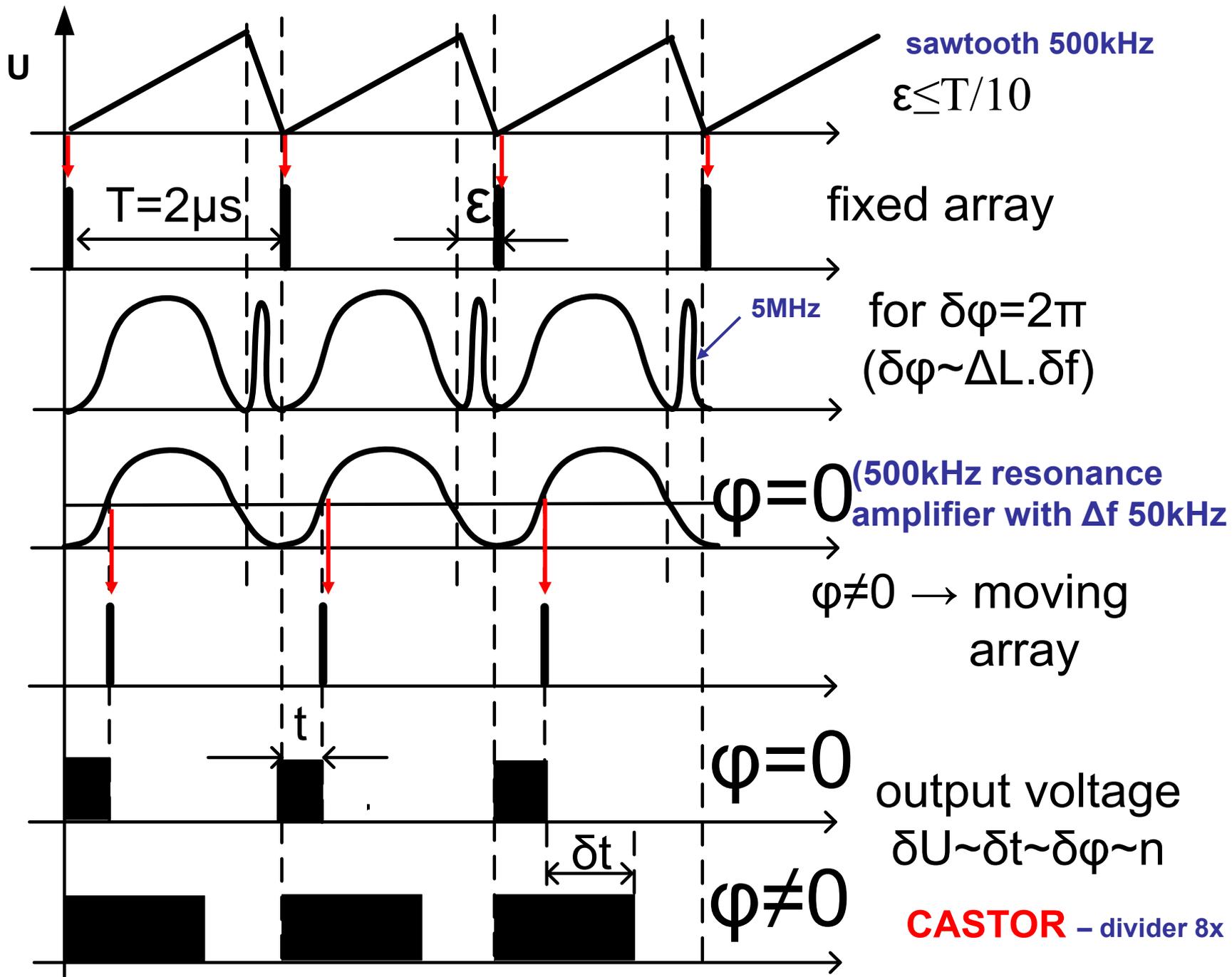
# Direct reading interferometer on CASTOR

tokamak

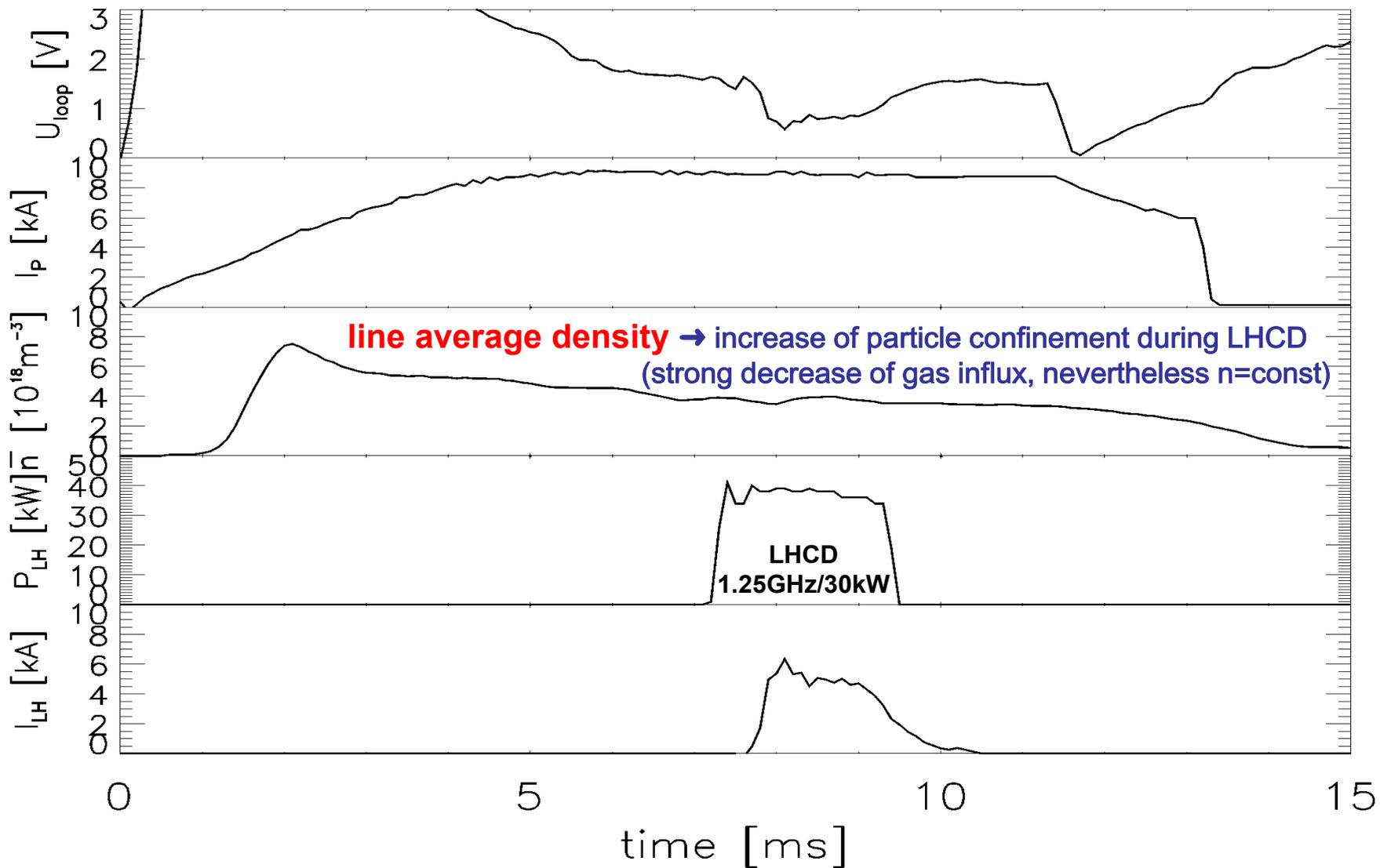
- homodyne,  
frequency modulated

$\omega = 2\pi \cdot 75\text{GHz}$  ( $\lambda = 4\text{mm}$ )  
generator





Loop voltage  $U_{loop}$ , plasma current  $I_p$ , **line averaged density  $n$** , incident LH power  $P_{inc}$  and non-inductively generated LH current  $I_{LH}$  in a typical CASTOR discharge with LHCD (shot #5581)

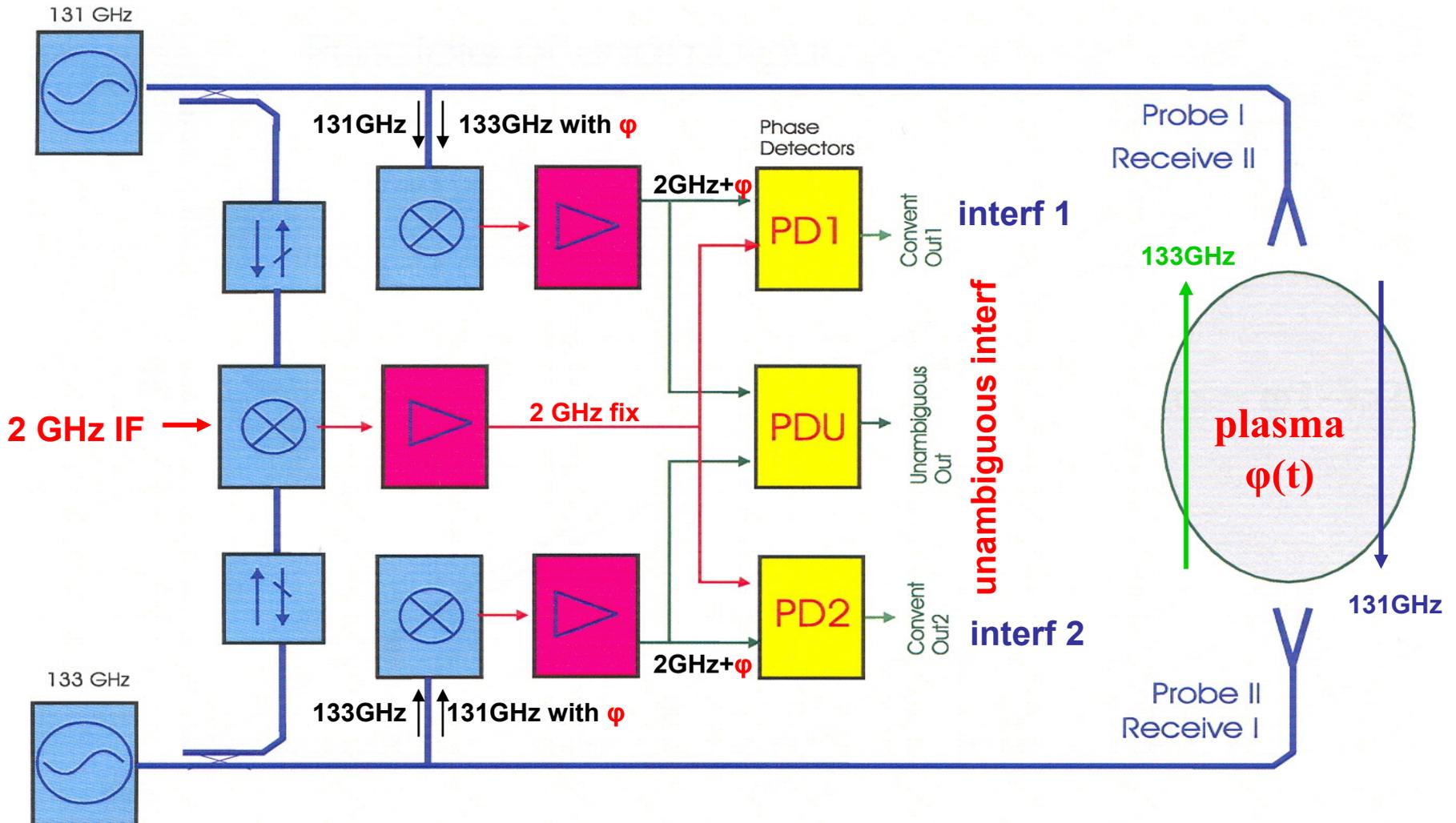


# Double (unambiguous) interferometer for tokamak COMPASS-D

(heterodyne system, where the **both waves** with constant frequencies are measuring waves)

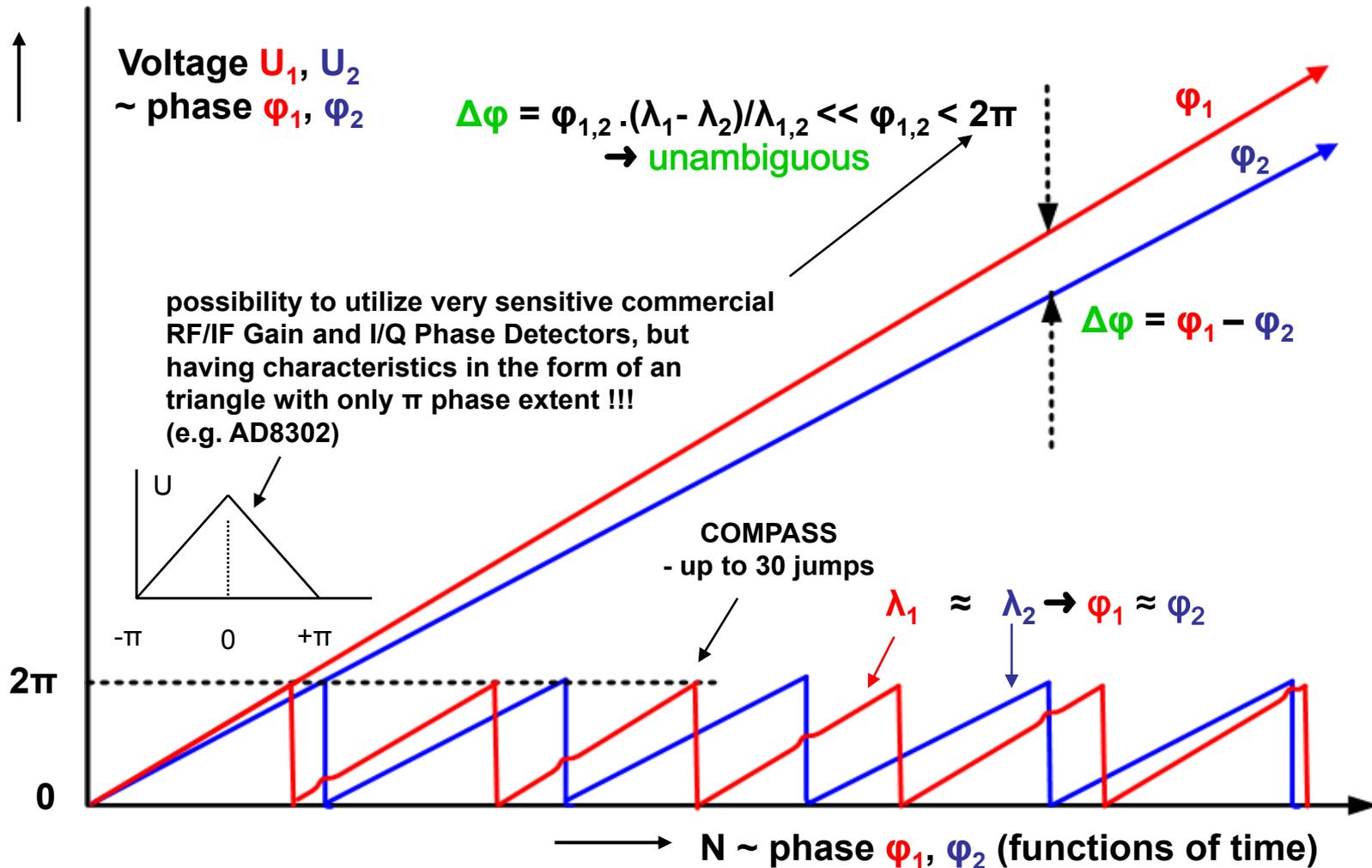
(see e.g. Annual Rep. of the EURATOM/UKAEA Fusion Programme 2000/01, p.108)

(will be used for feed-back control of the density averaged along diameter)

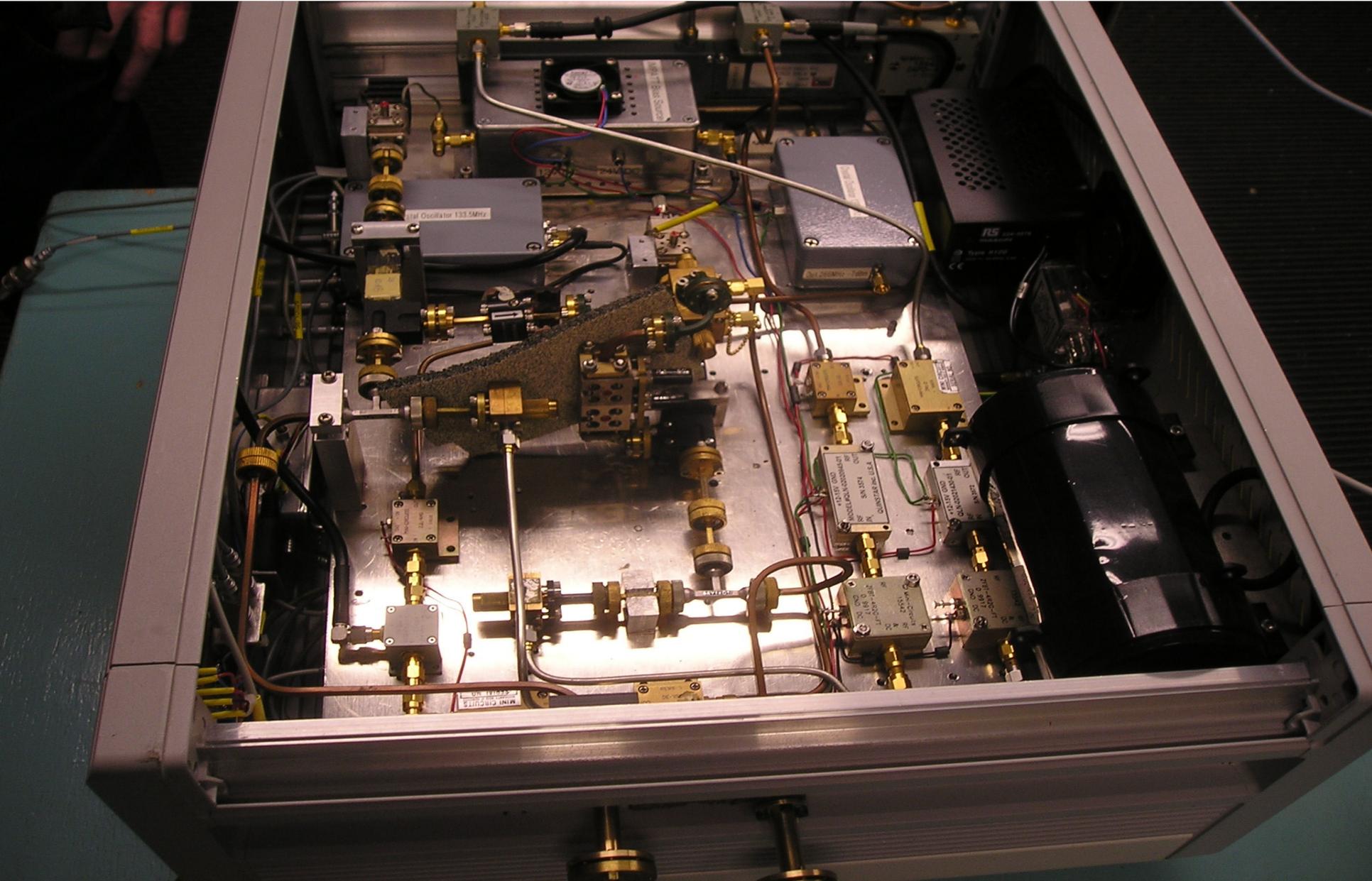


# Principle of **unambiguous** interferometer

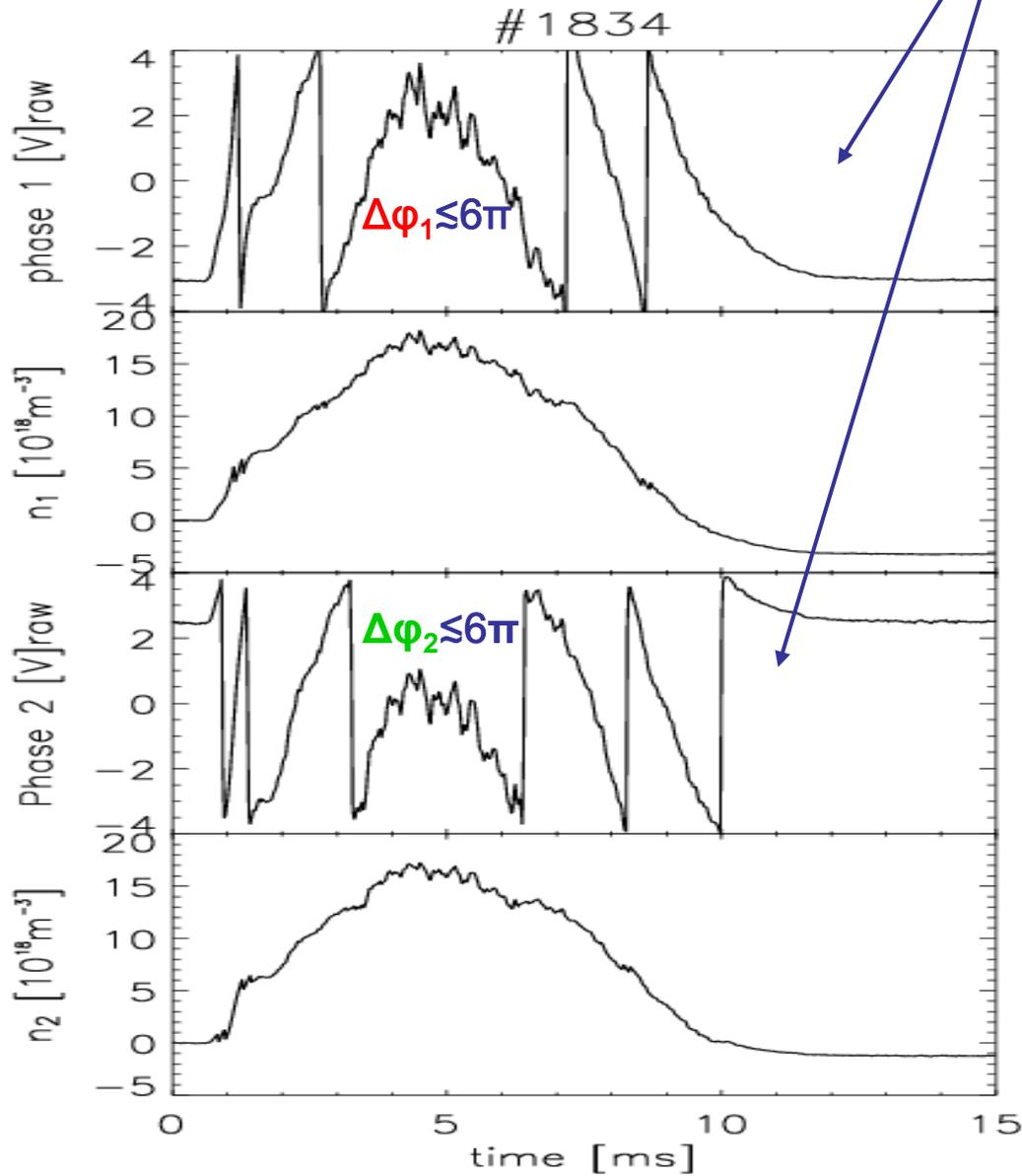
(with two probing waves with a little shifted, but fixed frequencies)



# Double (unambiguous) interferometer for tokamak COMPASS-D



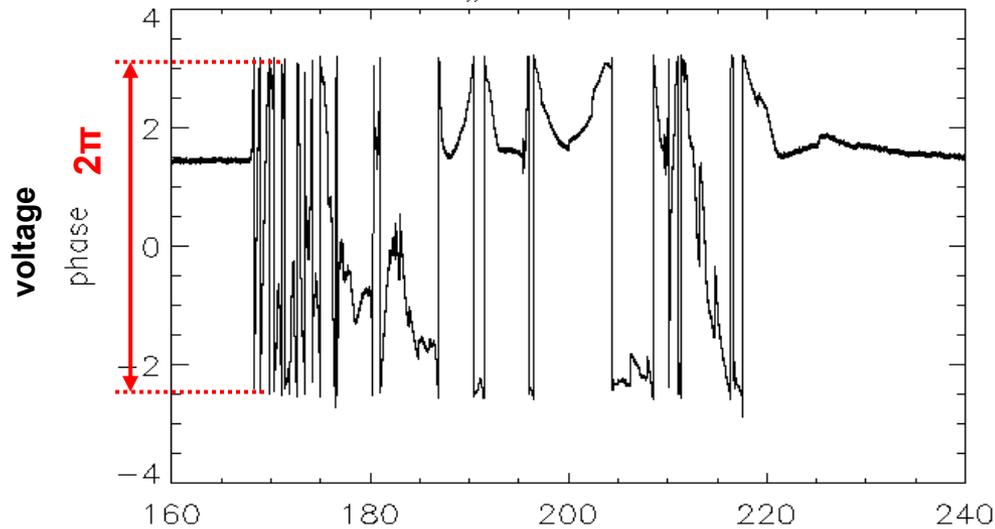
mutual phase shift  $\Delta\phi_1 - \Delta\phi_2$  changes  $2/132=65.5$  times slower than  $\Delta\phi_1$  or  $\Delta\phi_2$  itself  
 → unambiguous interferometer  
 (maximum shift never exceeds  $2\pi$ )



$\Delta\phi = 2\pi$

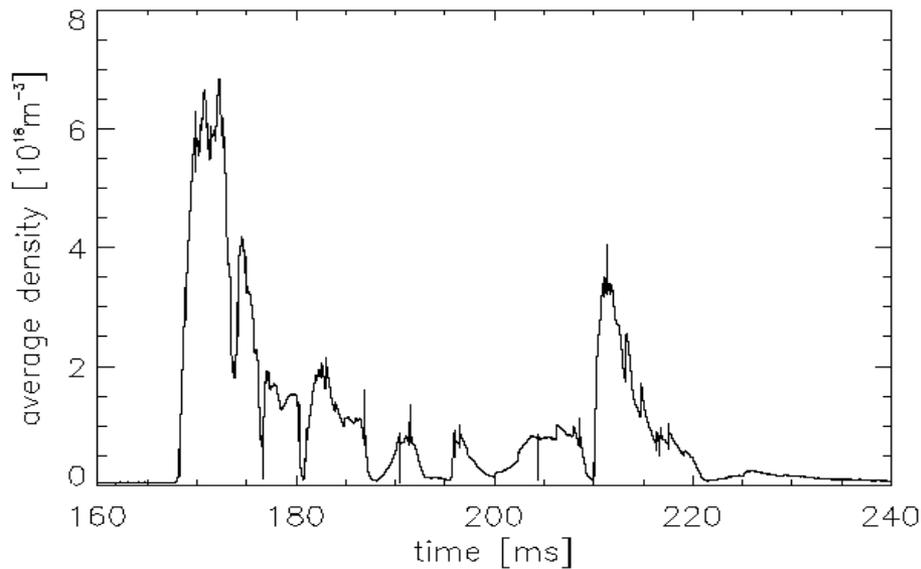
**CASTOR**  
 calibration of  
 double  
 interferometer  
**f=131 and 133 GHz**  
 (developed in Culham  
 for tokamak  
 COMPASS)  
 ( $\Delta\phi = 2\pi \Rightarrow 8\text{V}$ )

COMPASS #208, Dec 10, 2008



## COMPASS line averaged density

start of operation, Dec 10, 2008  
(without plasma position feed-back)



evaluated from only one interferometer

by SW program under two conditions:

a) if  $\varphi_{i+1} - \varphi_i < -\pi$ , then  $\varphi_{i+1} = \varphi_i + 2\pi$

b) if  $\varphi_{i+1} - \varphi_i > +\pi$ , then  $\varphi_{i+1} = \varphi_i - 2\pi$

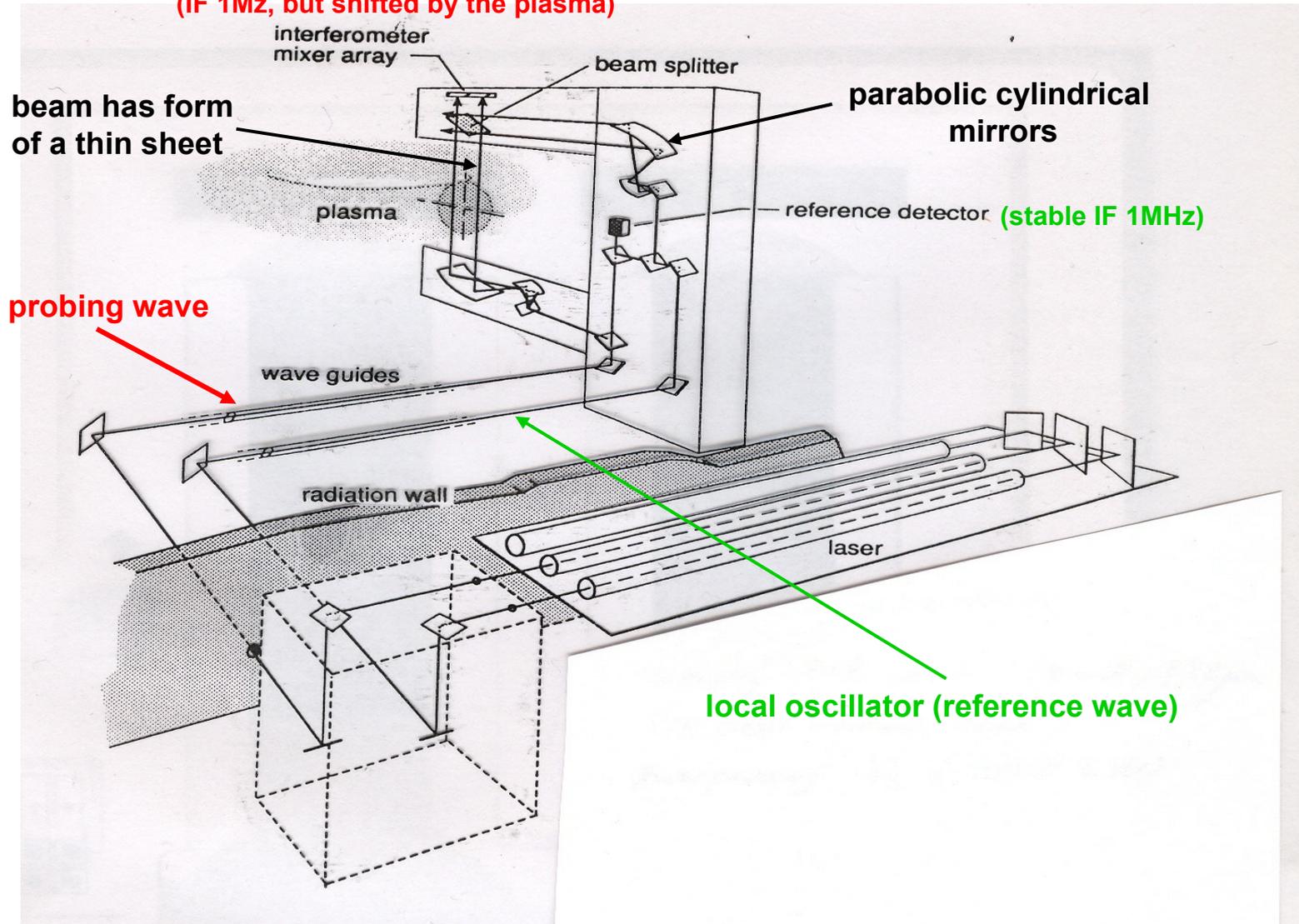
# RTP tokamak - double FIR interferometer ( $\Delta f = 1$ MHz)

2x50mW/432 $\mu$ m (HCOOH – formid acid)

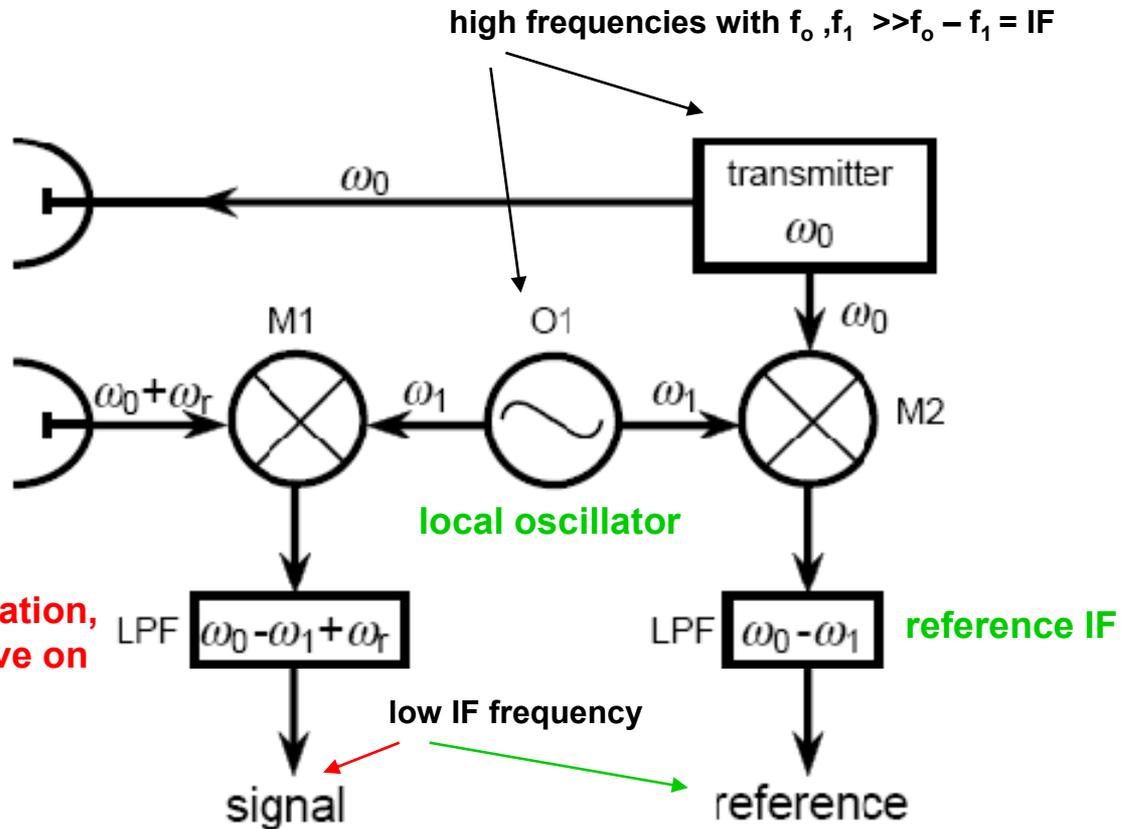
optically pumped by IR CO<sub>2</sub> laser (150W CW)

(heterodyne system, where **only one wave is measuring**)

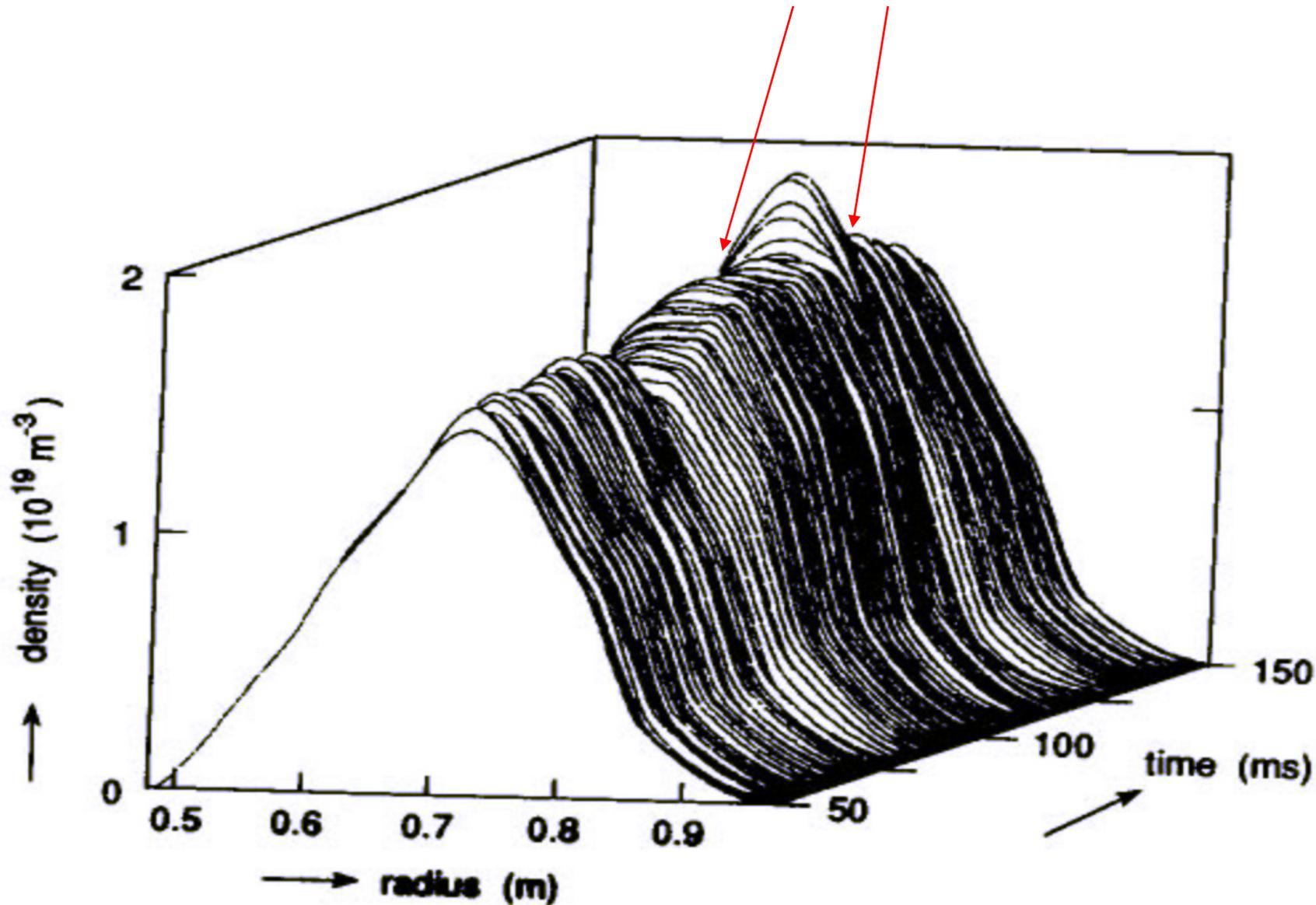
(IF 1Mz, but shifted by the plasma)



# Principle of heterodyne detection



**Time development of the density profile for ECRH heated RTP discharge**  
(RF is switched on from 80 to 121ms)



# Sin – cosin reflektometer on CASTOR

(X-mode, 35GHz/200mW,  
resolution hundreds of kHz )

$$E_m = E_m(t) \cdot \sin[\omega t + \varphi(t)]$$

$$E_r = E_r \cdot \sin \omega t$$

quadratic diodes,  $\varepsilon(t) = E_m(t)/E_r$ :

$$U_1(t) = E_r^2 (1 + \varepsilon^2(t) + 2\varepsilon(t) \cos \varphi)$$

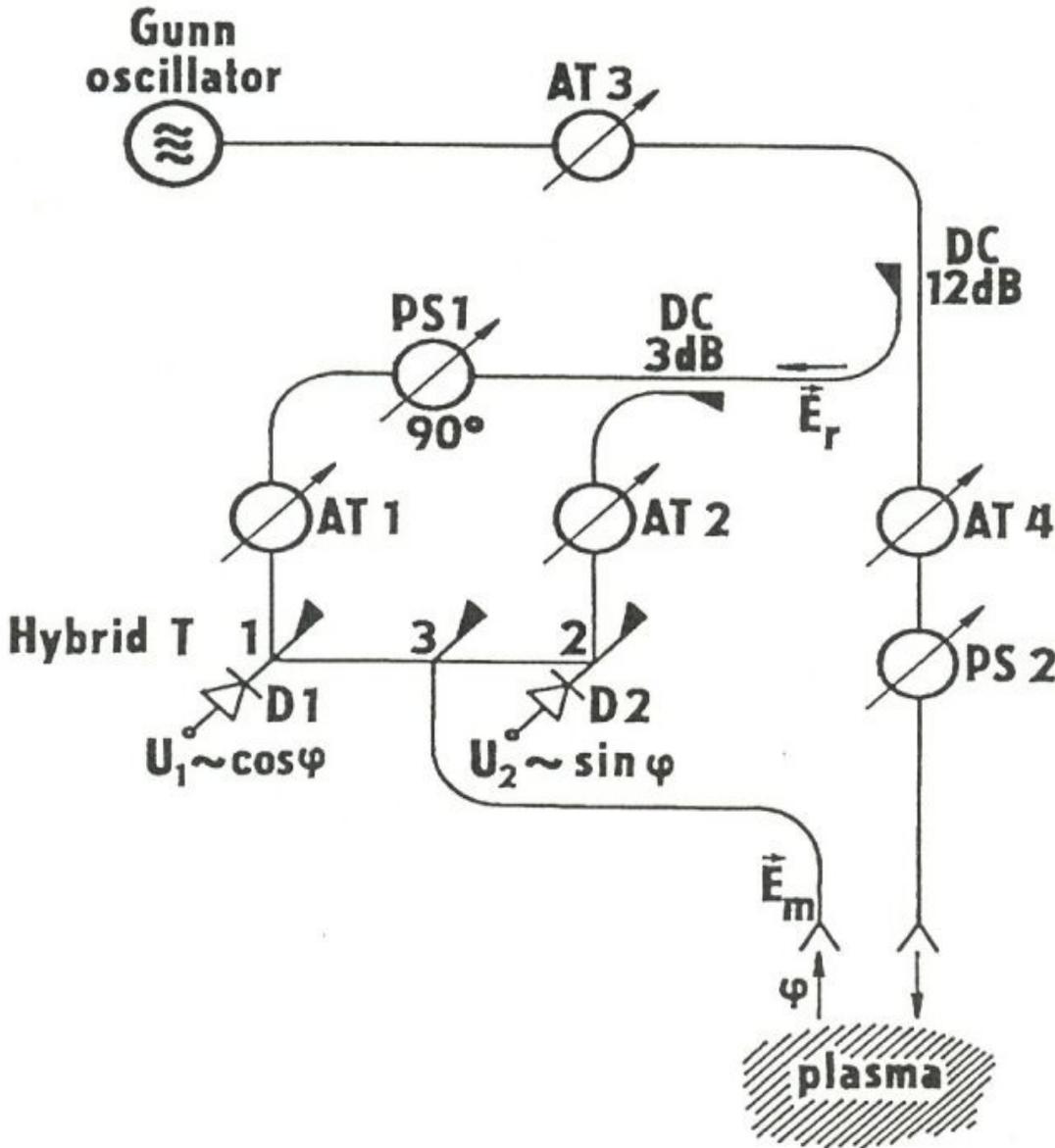
$$U_2(t) = E_r^2 (1 + \varepsilon^2(t) + 2\varepsilon(t) \sin \varphi)$$

assumption:

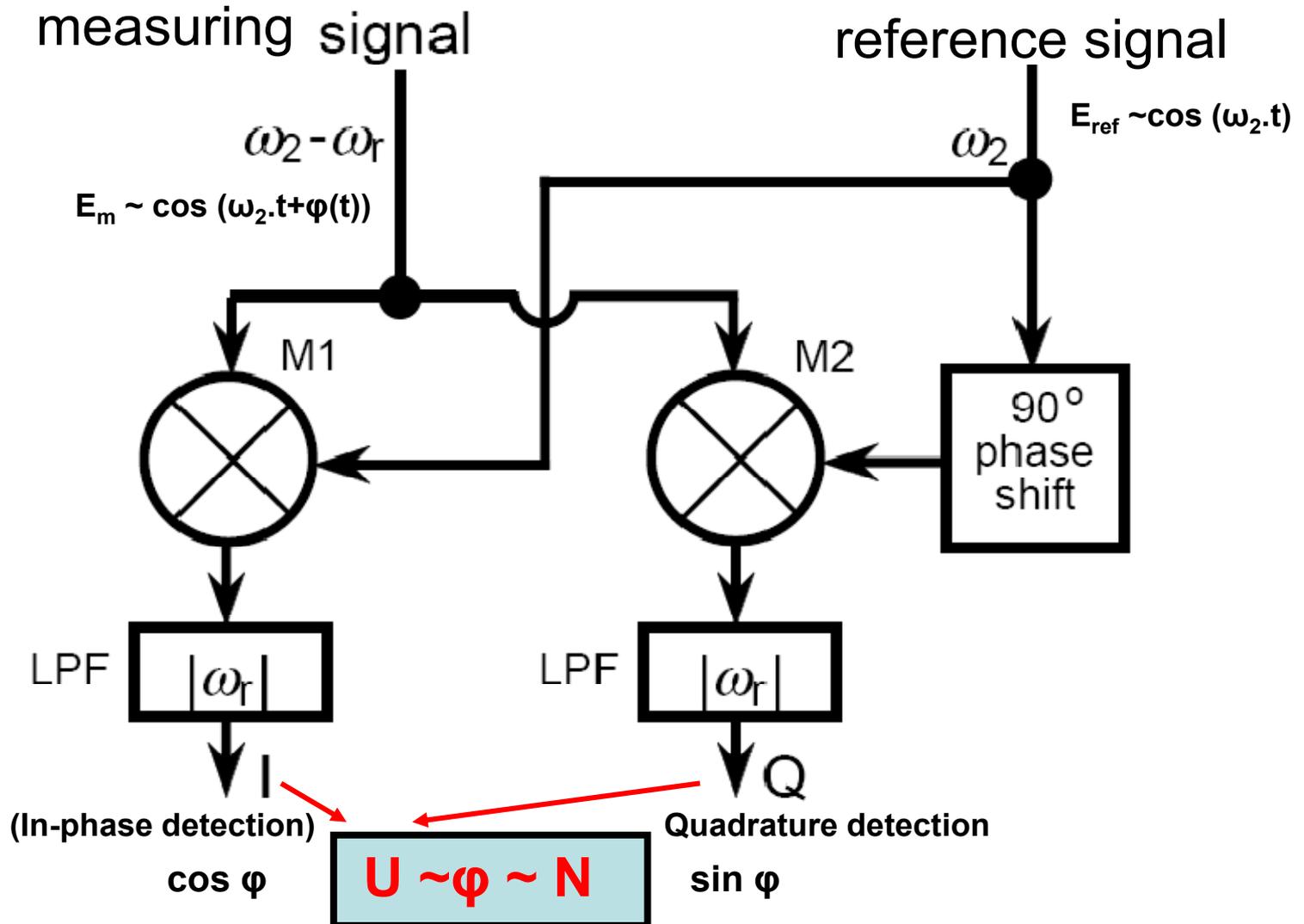
1. diodes are identical
2. diodes are quadratic
3.  $E_m \ll E_r$  (i.e..  $\varepsilon^2 \ll 1$ , e.g. for -20db the error  $\delta\varphi = 4^\circ$ )

then **without AC component** we obtain:

$$\varphi(t) = \arctg(U_2(t)/U_1(t))$$

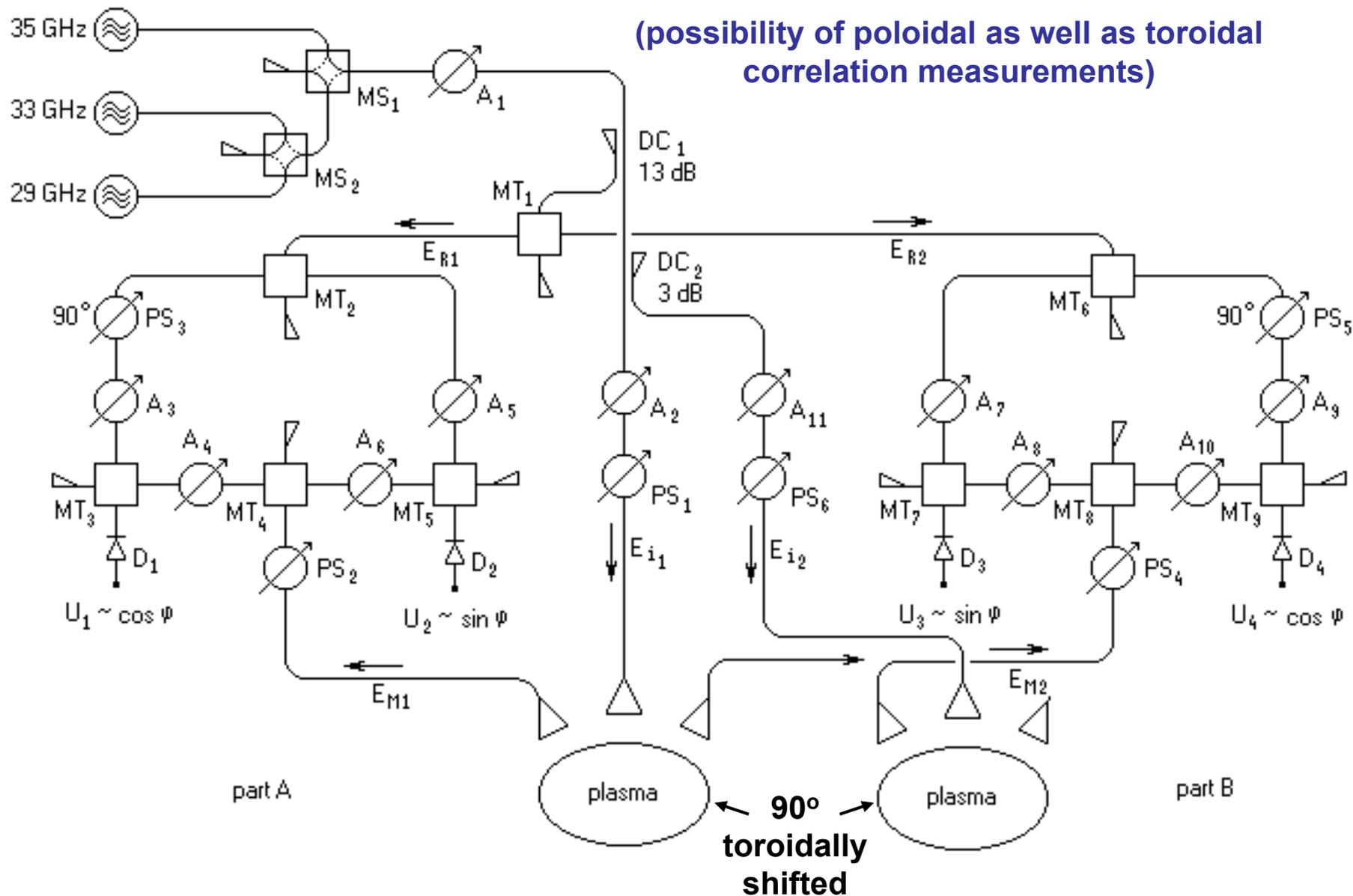


# I/Q phase detection scheme $(\varphi(t) = \int \Delta\omega \cdot t' = \int \omega_r \cdot t)$ using a commercial electronics

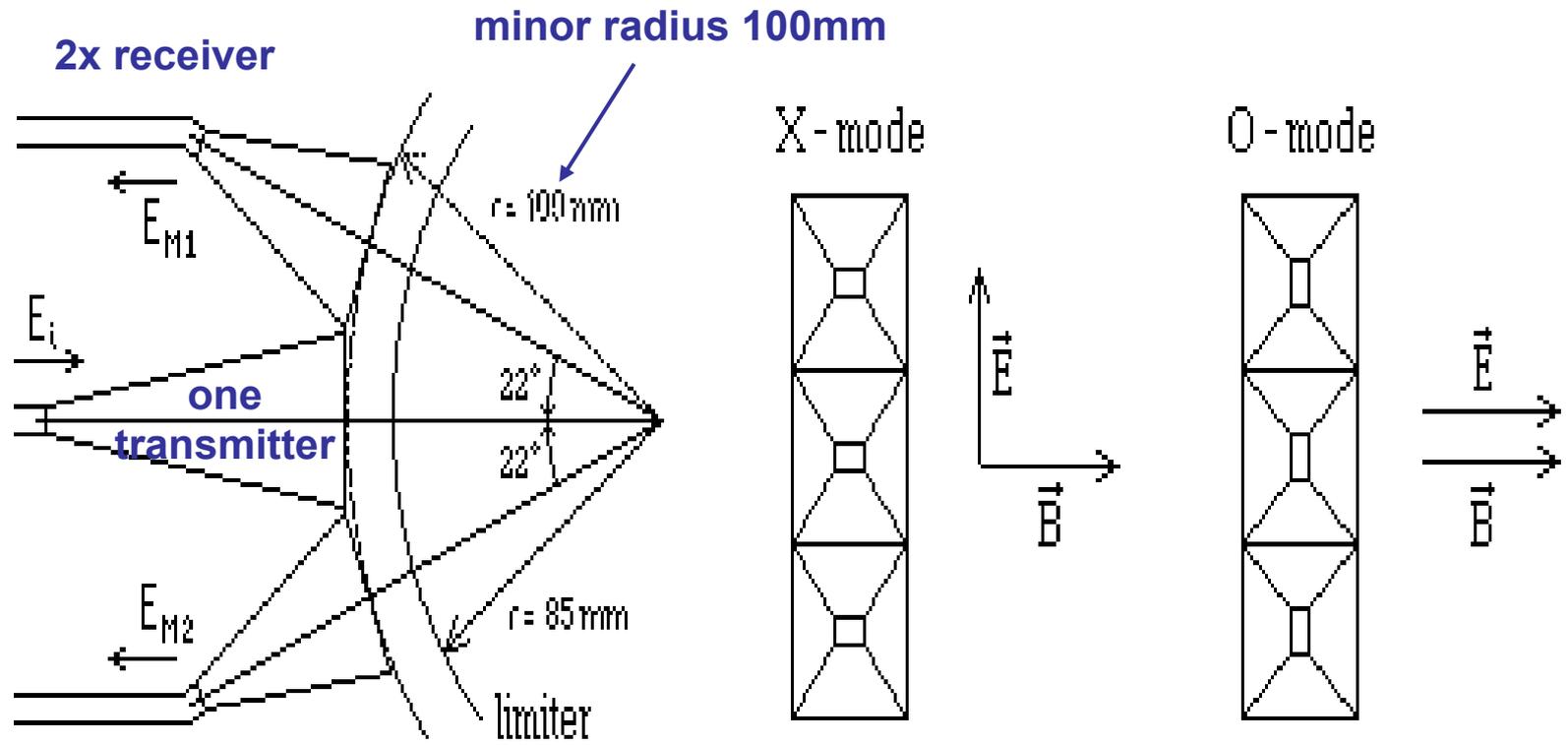


# Three-frequencies reflectometer on CASTOR

(possibility of poloidal as well as toroidal correlation measurements)

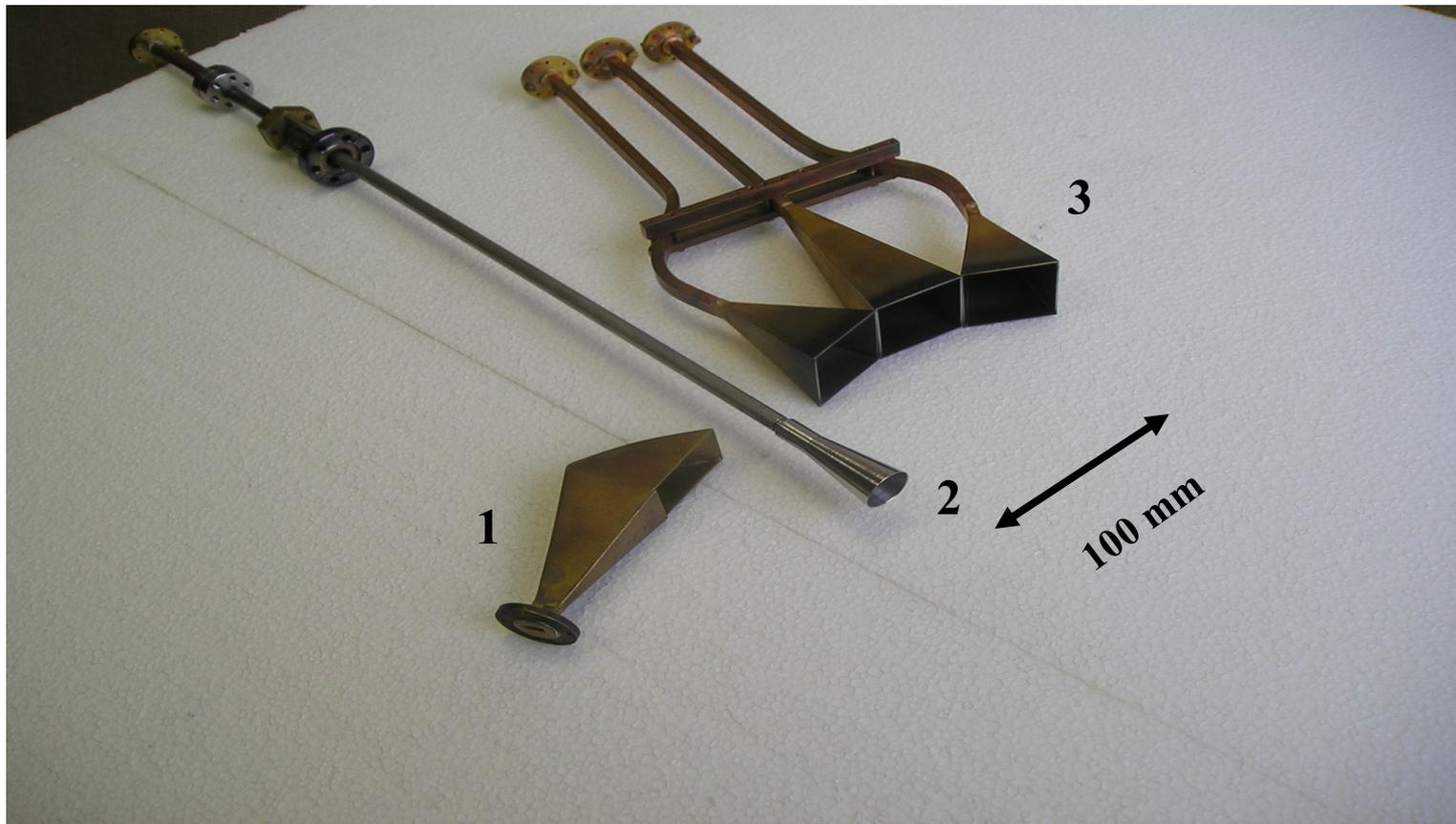


# Antenna system of reflectometer used on tokamak CASTOR



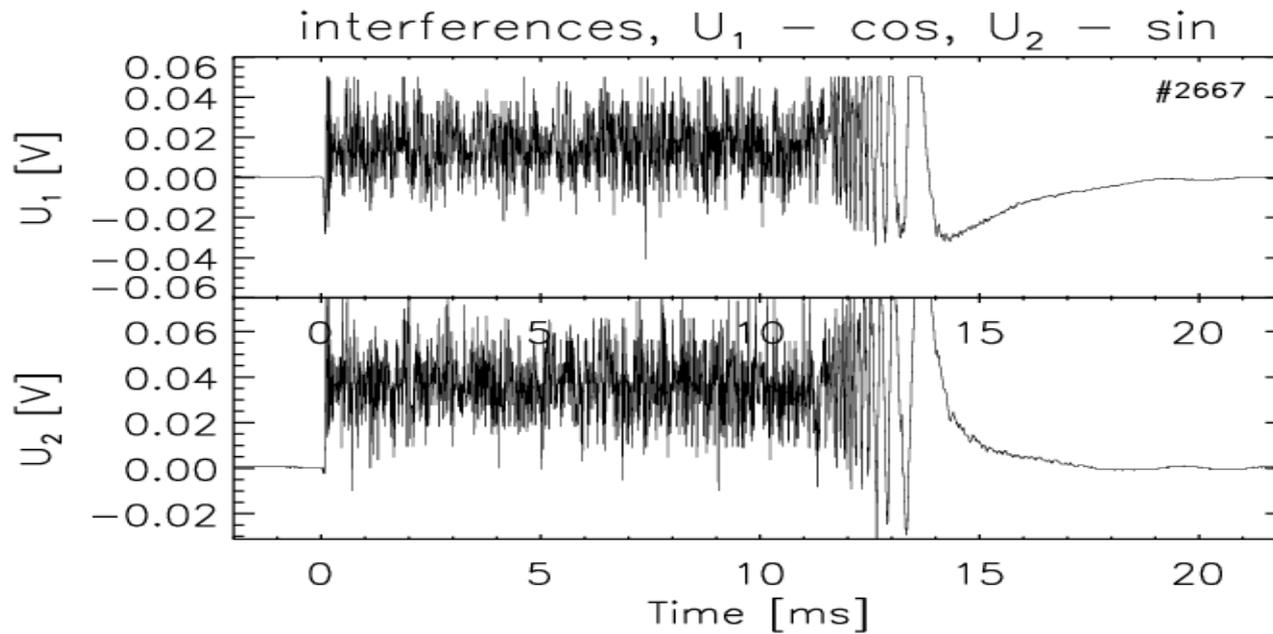
## **CASTOR – 8mm microwave diagnostic antennas:**

- 1 ... rectangular hog-horn for HFS measurements**
- 2 ... circular for 4mm interferometer**
- 3 ... system of three rectangular antennas of reflectometer working in v O-mode**

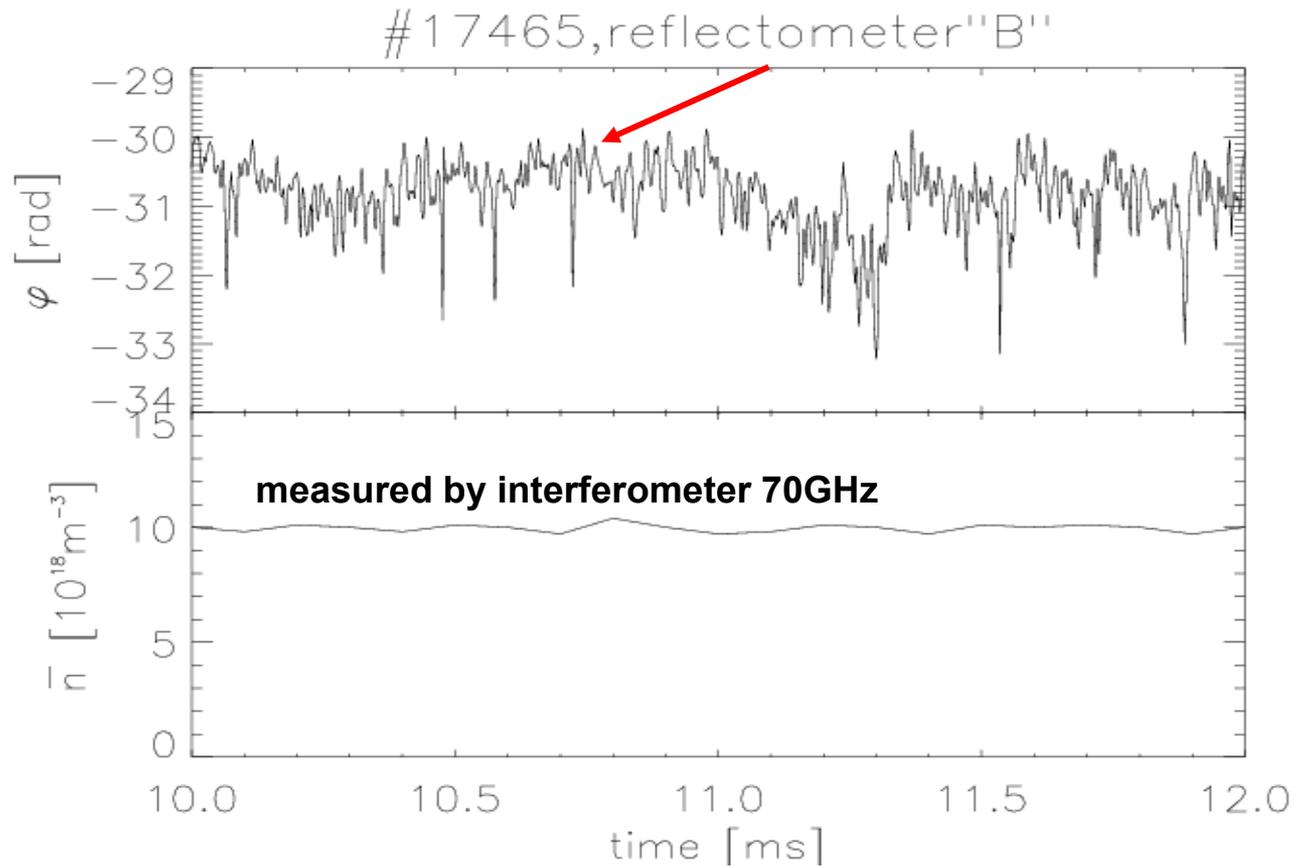


# Three-frequencies reflectometr on CASTOR



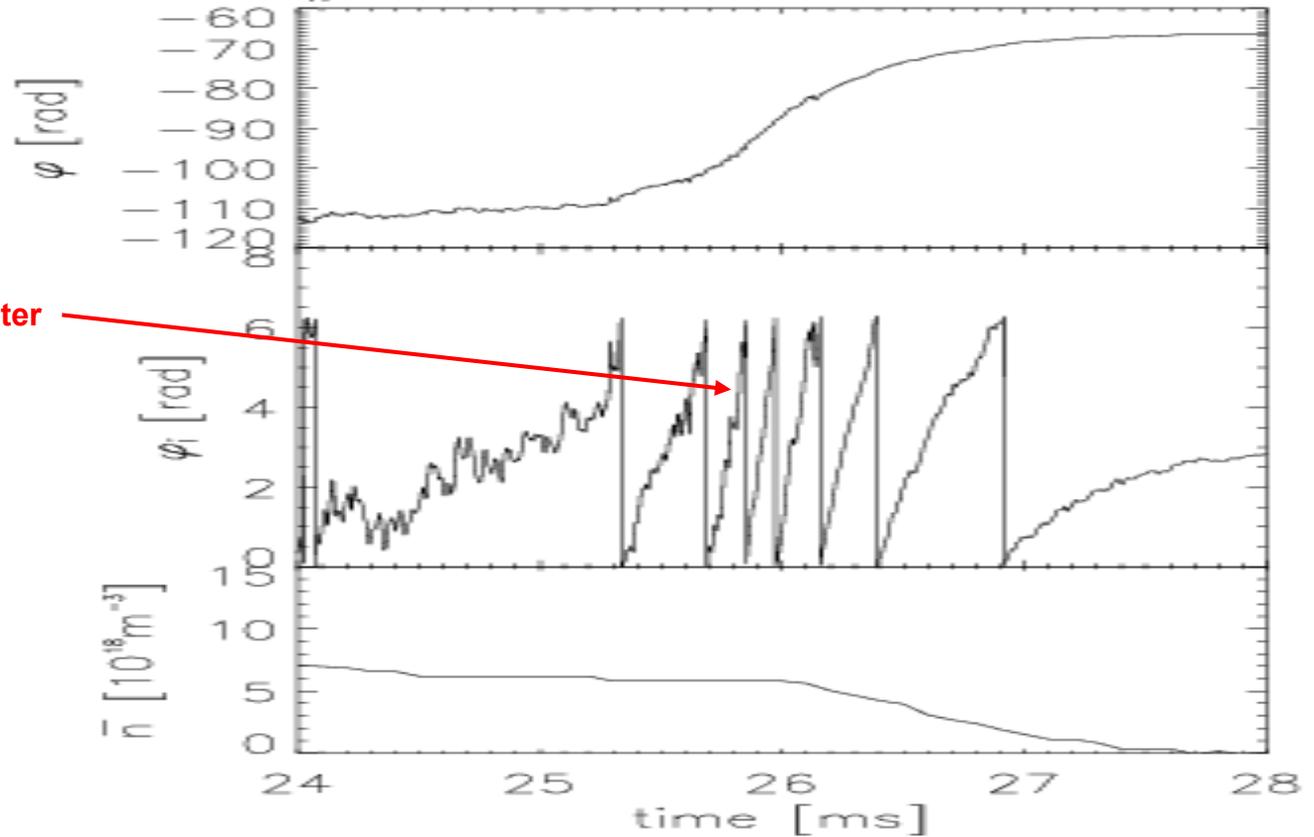


Record of the **CASTOR** 33GHz reflectometer signal  
(AC input)



**Phase fluctuations of the reflected wave during the stationary phase of tokamak CASTOR discharge**  
( $\delta n/n$  from measured  $\varphi$ , depending on the gradient  $n$  supposed in the place of reflection, has been estimated to be about 2-5%)

#17465, reflectometer "A"

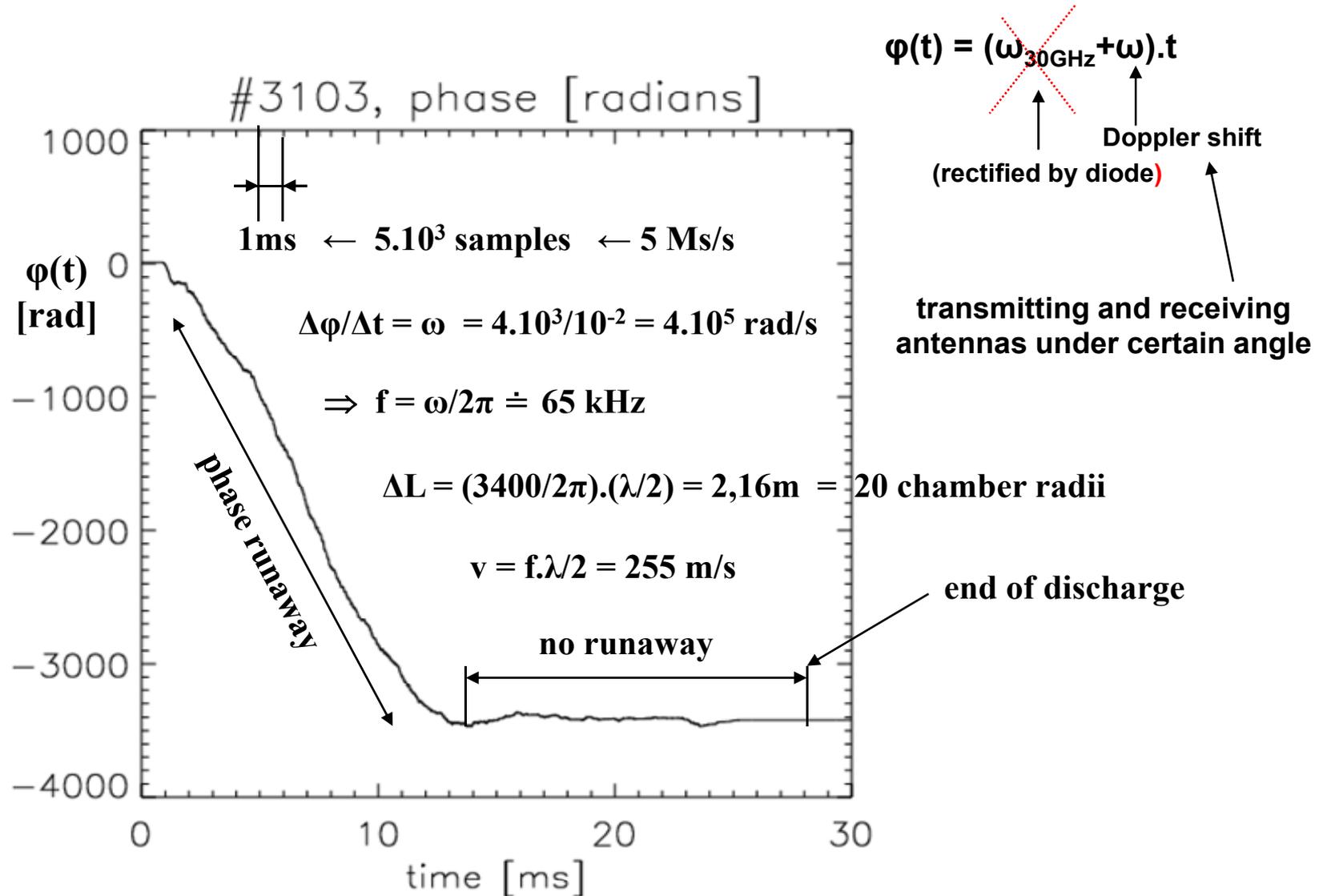


double-pass interferometer  
on 8mm (reflection on  
the tokamak HFS wall)

**Reflectometry phase measurement  
in the end discharge phase**

# CASTOR – „Phase runaway“ phenomenon, result of Doppler shift of reflected signal by plasma turbulence

(Zacek F., ...: 18<sup>th</sup> SPPT, 1997, 42)



# Principle of fluctuating plasma density profile measurement using a method of fast swept reflectometry

**Principle** of **classical reflectometry** – measurement of the phase  $\varphi$  of the reflected wave (by interfering with the reference wave with  $\varphi_0$ ):

1. if generator frequency  $f = \text{const}$  (homodyne or heterodyne reflectometry) and reflection occurs on a fixed metal plane, then  $\varphi - \varphi_0 = \text{const}$  and also interference signal  $U = U(E_{\text{ref}}, E_m, \varphi) = \text{const}$ , but periodically depending on  $\varphi$ , i.e. on the reflection plane distance  $L$
2. if frequency  $f$  is swept ( $f$  has a saw tooth form in a certain frequency range), i.e.  $f = k.t$  and reflection occurs on the fixed mirror in a distance  $L$ , then  $\varphi = \text{const}.t$  and interference signal  $U$  will show beats  $\sim \sin(\omega_b.t)$  with constant frequency  $\omega_b = 4.\pi.(L/c).df/dt$

**Important:** Beat frequency is function of two parameters:

- distance  $L$  of the reflection point
- and speed of the sweeping  $df/dt$  *only* (it is not function of the frequency  $f$  itself !!!).

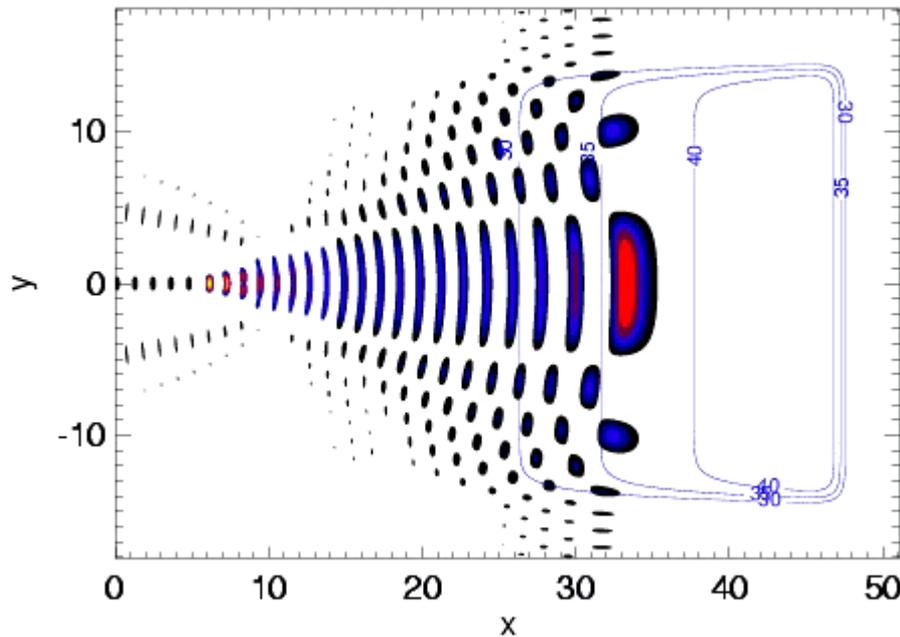
**Note:** For  $L$  determination (commercial FMCW radars),  $df/dt = \text{const}$  is needed (high linearity  $f$  with time must be assured)

3. In **plasma** is the situation concerning fast swept reflectometry much more complex - from two reasons:
  - a) the **point of the reflection** in the plasma, i.e. also  $\omega_b$ , **depends on the actual wave frequency**  $\rightarrow L$  is increasing with increasing frequency and just this effect (increase of  $\omega_b$  by  $\Delta\omega_b$ ) can be used for the profile measurement (through localization of the reflection point in the corresponding density)
  - b) **but, due to the density  $\delta n$  fluctuations**, the point of the reflection  $L$  (even at  $f = const$  and density  $\langle n \rangle = const$ ) is not localized all the time on the same radius, but it is fluctuating together with density fluctuations in the place of the wave reflection – this **disturbing effect** results also in broadening of  $\omega_b$  (let us denote it by  $\delta\omega_b$ )
  
4. The problem is, how to **eliminate that undesirable effect  $\delta n$  of density fluctuations** on  $\omega_b$  and so how to determine that searched  $\Delta\omega_b$  ;  
it is enabled by **statistical correlation methods** of the receiving signal (sampling rate tens of MHz is needed, nevertheless „sliding FFT“ needed for the signal frequency analysis has windows longer than they are typical plasma fluctuations and a „proper“  $\Delta\omega_b$  must be therefore found from “time history” of the measurement

# Effect of fluctuations on reflected electric field

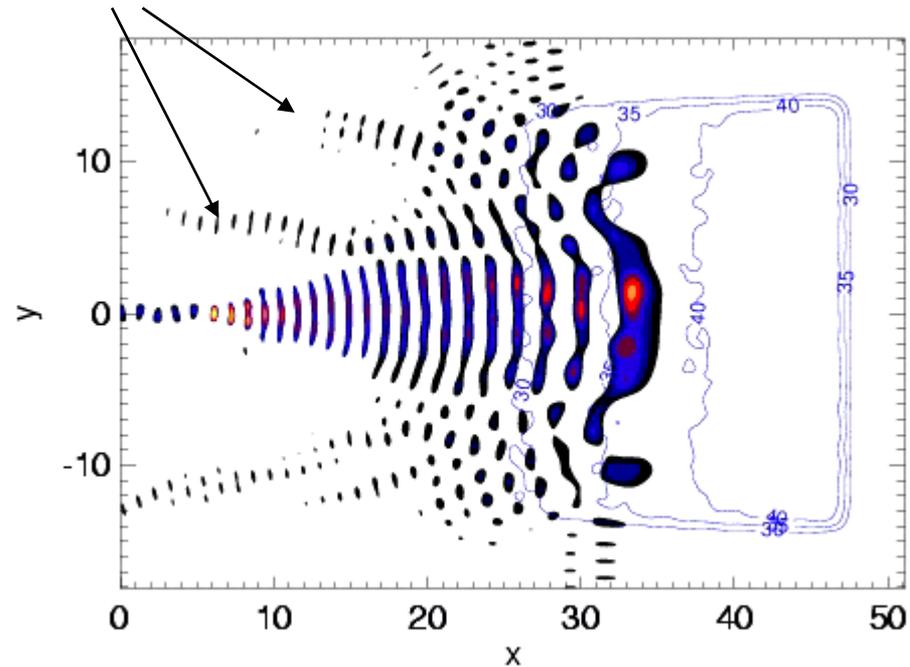
(taken from a lecture of M. Manso, Association EURATOM/IST, Lisboa, Portugal)

no turbulence



with turbulence

fluctuating side lobes (wave front)



→ the plasma density perturbations strongly distort the spatial electric field pattern causing scattering of probing wave energy.

# Comparison of impulse radar and fast swept reflectometry

- **classical impulse radar** - time of the short pulse return, after its reflection on the objection, is measured (pulse propagate with **group velocity**, which equals phase velocity only in non-dispersive medium)
- by application for detection of small distances has the **classical radar (= pulse reflectometry)** problems with the time (and therefore also with the space) resolution, due to the non zero pulse length
- **reflectometry with the fast frequency sweeping** is nothing else than **CW radar** (commercial products are also called **FMCW radar**), because generator transmits wave **continuously** and the phase of the wave reflected on the objection is continuously detected (the phase propagates with **phase velocity**)
- if the wave enter plasma at  $x=x_0$  and the critical density is located at  $x=x_c(\omega)$ , then the **phase  $\varphi$  of the reflected wave** after its return can be written as follows:

$$\varphi(\omega) = 2(\omega/c) \cdot \int_{x_0}^{x_c(\omega)} n(\omega, x) \cdot dx + \varphi_0(\omega)$$

- it may be easily shown that in inhomogeneous plasma **the frequency derivative** of the **phase**  $d\varphi(\omega)/d\omega$  of the reflected wave (frequency, not the time derivative) **equals the total group delay** of the wave  $\tau_g$  propagating from the plasma edge  $x_0$  to the point of the critical density  $x_{crit}(\omega)$  and back, i.e.:

$$d\varphi(\omega)/d\omega = \tau_g = \int_0^{\tau_g} dt = 2 \int_{x_0}^{x_{crit}(\omega)} dx/v_{gx} ,$$

and therefore,  **$d\varphi/d\omega$**  determines the actual distance of the **reflected point** (i.e. in tokamaks radial position of the frequency corresponding density).

- so, the **group velocity**, not phase velocity, is directly measured
- much more complicated interferometric measurement of the phase wave  $\varphi$ , which can be moreover “interrupted”, must not be performed because a **direct output of the measurement** – an analysis of the beat frequency behavior – can be used.
- a small departure  $\Delta\omega_b$  with frequency increasing in time, caused by dependence of the reflection point in the plasma on the frequency (in the case of reflection on a fixed mirror would be  $\omega_b = const$ ), must be distinguished from  $\delta\omega_b$  caused by density fluctuations).

# Radial profiles of frequencies in tomamak AUG

$f_p$  ... O wave cutoff

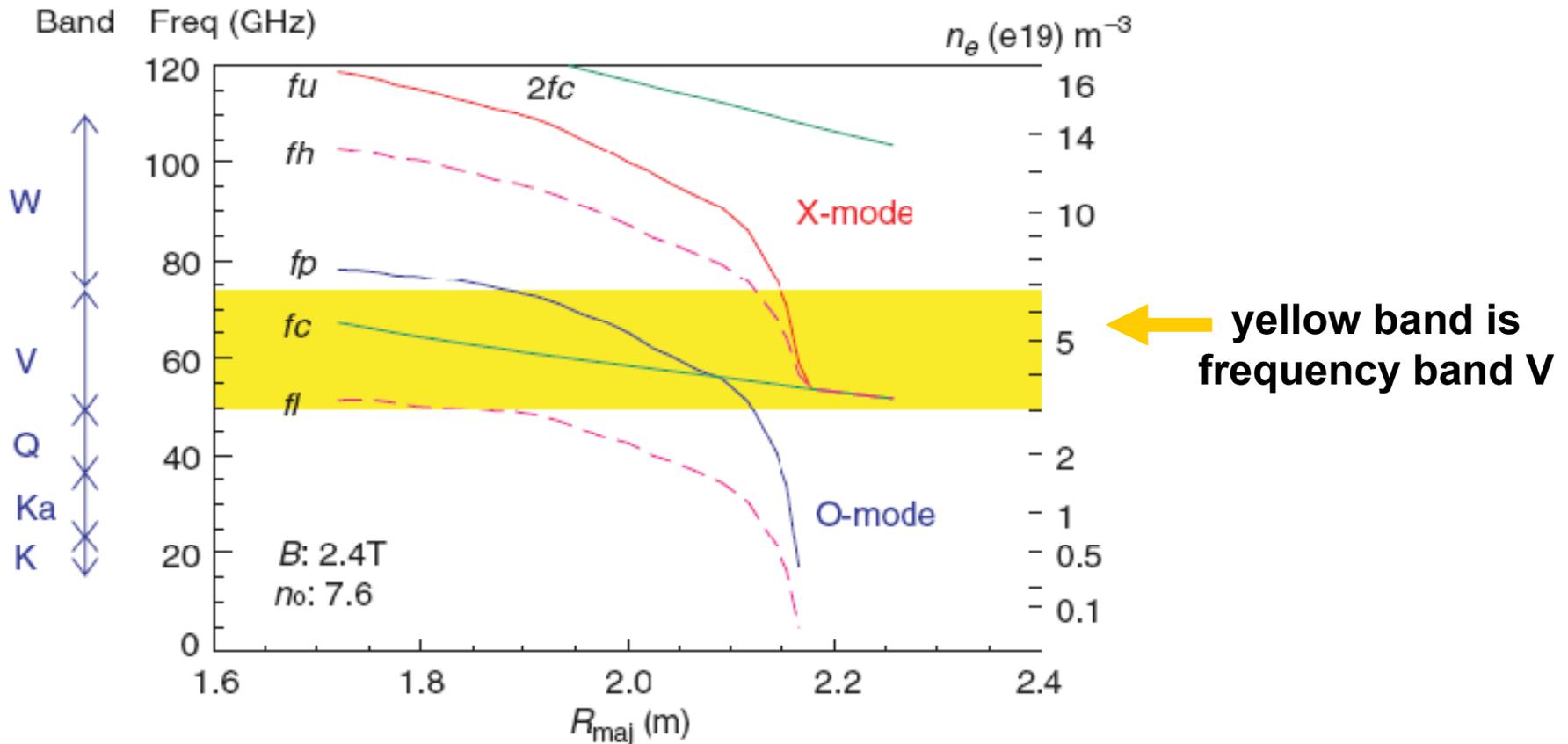
$f_u, f_l$  ... X wave cutoffs

$f_h$  ... upper hybrid resonance

$f_c$  ... electron cyclotron frequency

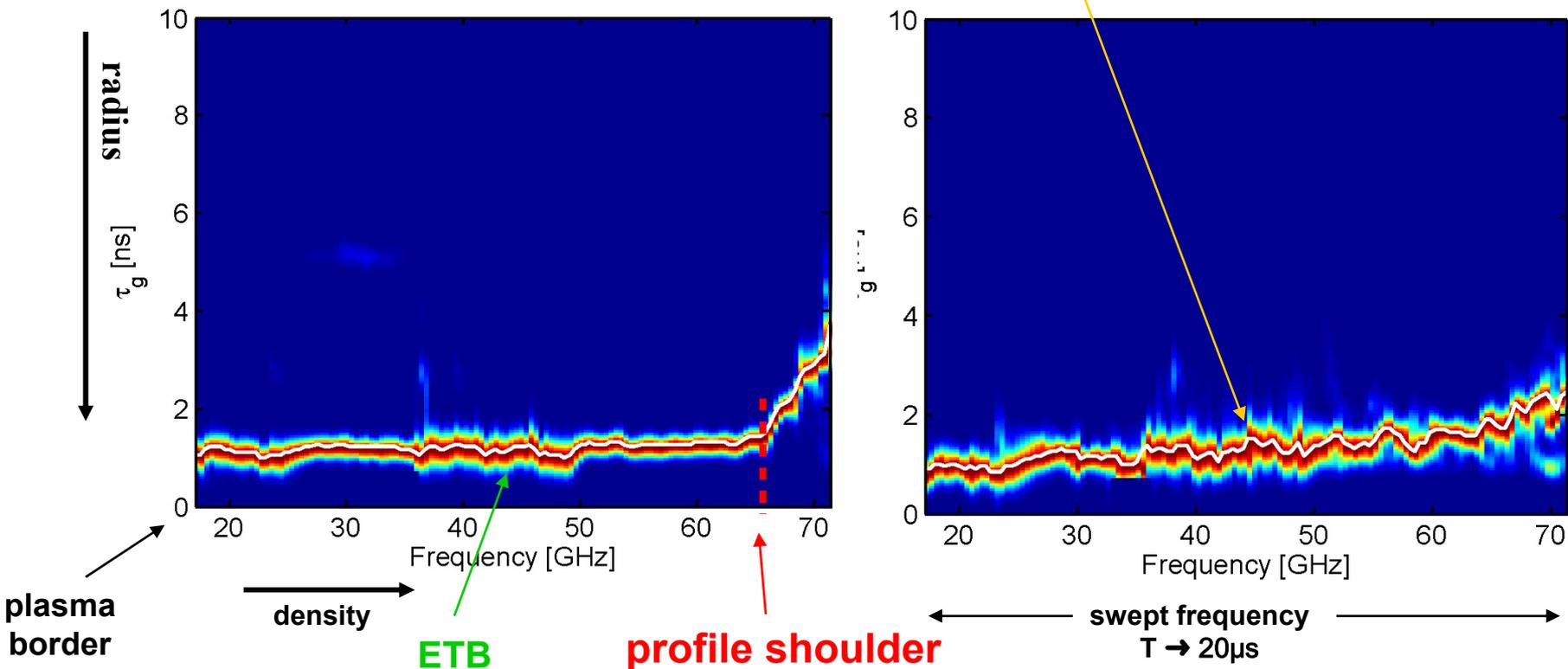
magnetic field on the device axis 2,4T

density on the device axis  $7,6 \cdot 10^{19} \text{m}^{-3}$  (profile determined by TS)



# ASDEX Upgrade – broadband FM-CW reflectometry - observation of ELMs

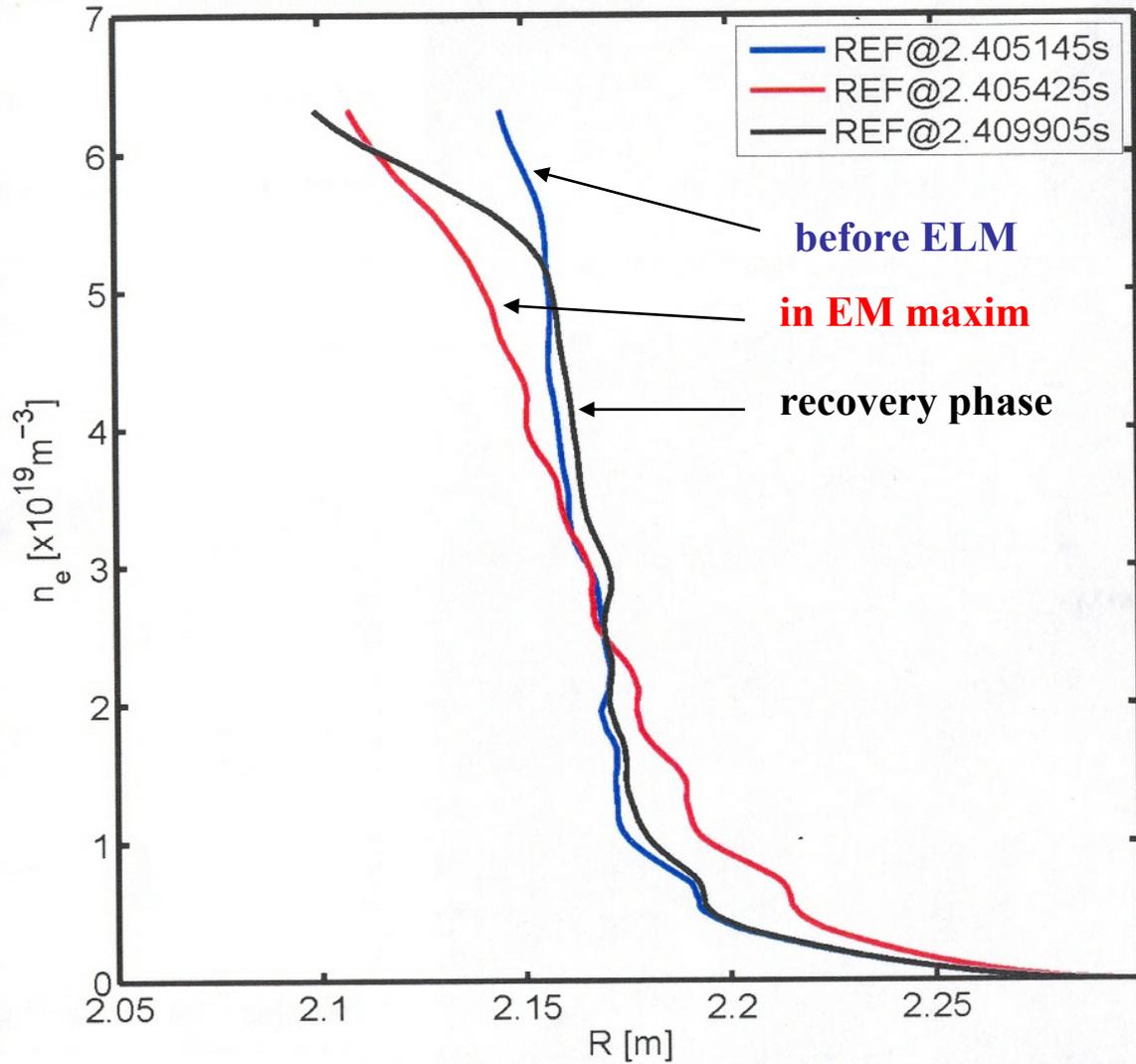
effect of density **fluctuations**  
(the same frequency reflects on a slightly different radius)



• Between ELMs

• During ELMs

#19911



# ASDEX-UG reflectometric measurement of density profile

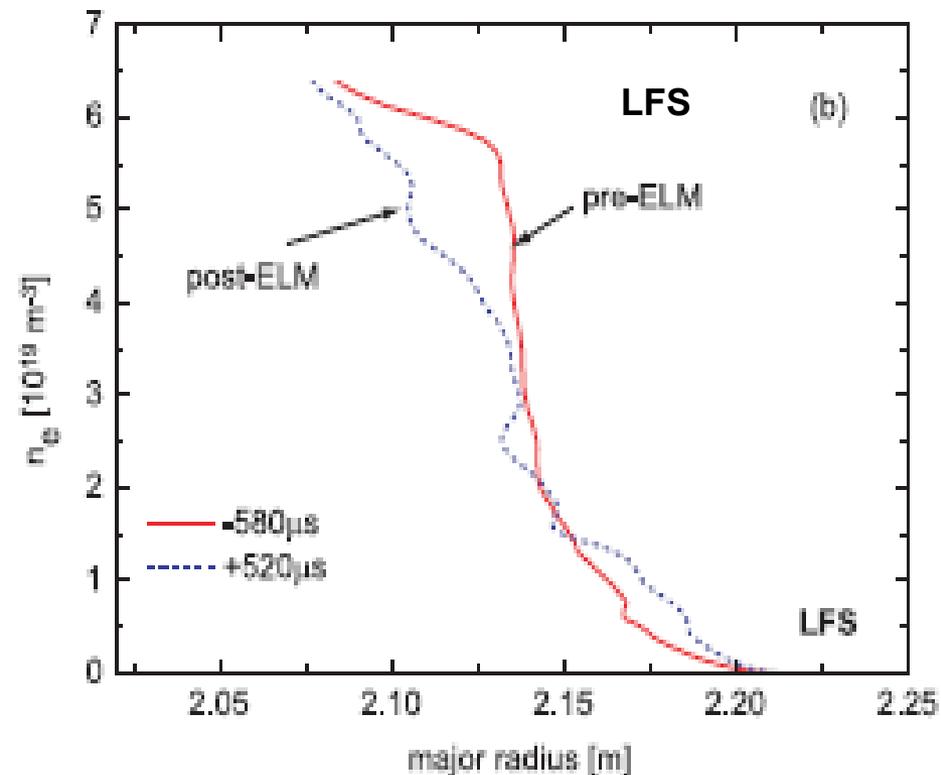
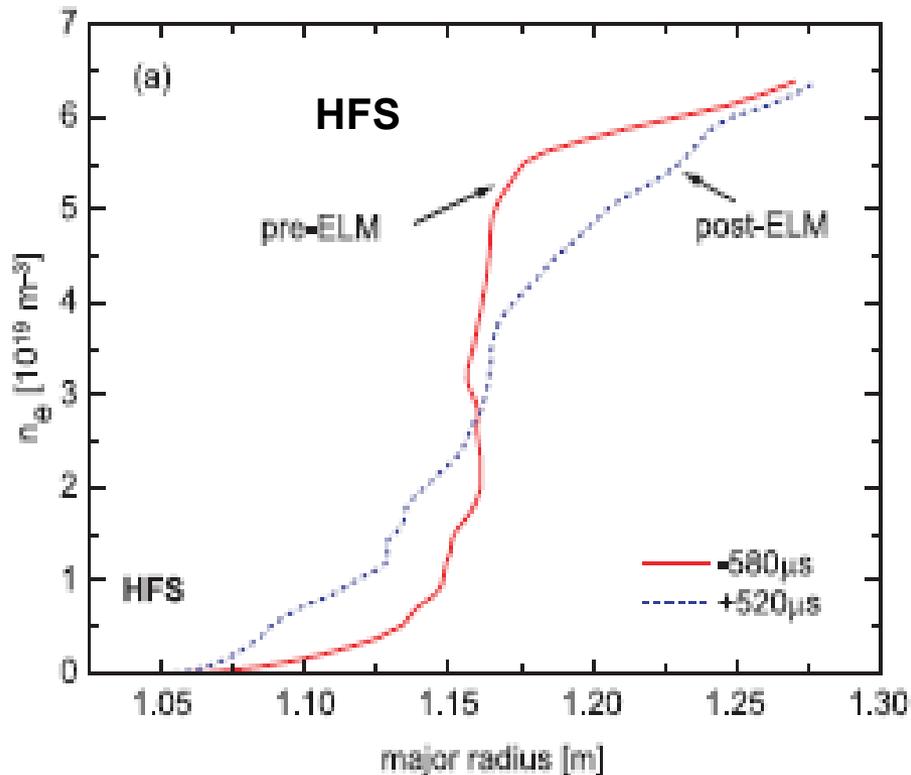
in different ELM phase

# ASDEX-UG, reflectometry measurement of HFS and LFS density profiles before (red) and after ELM (blue)

ELM start is put in  $t=0$

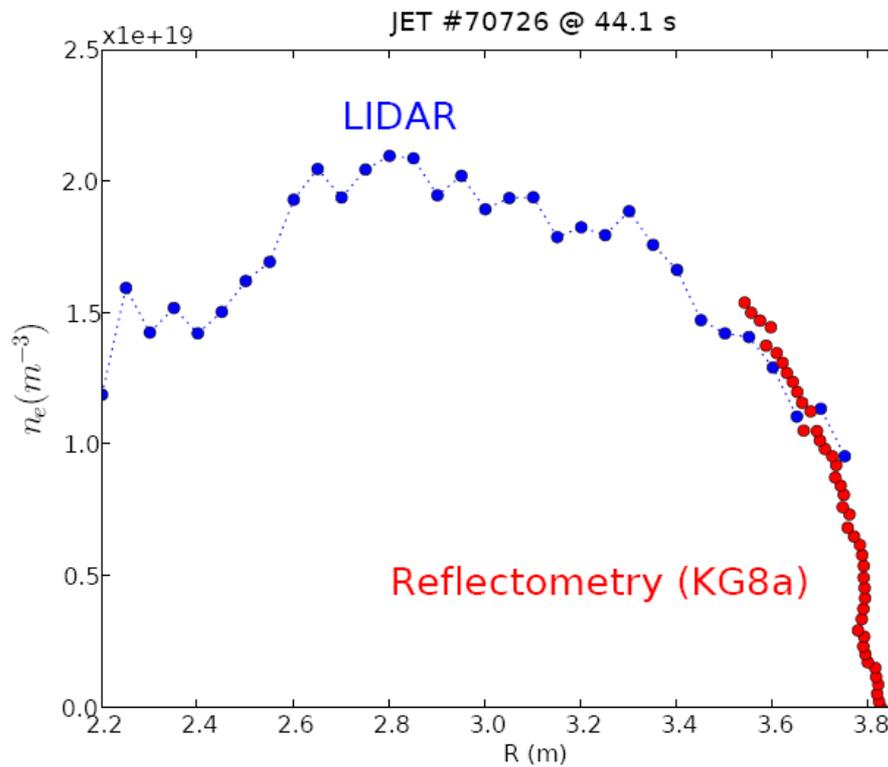
after ELM start a collapse of density profile occurs:

- region of pedestal steps back inside the chamber
- SOL broadens towards HFS as well as LFS

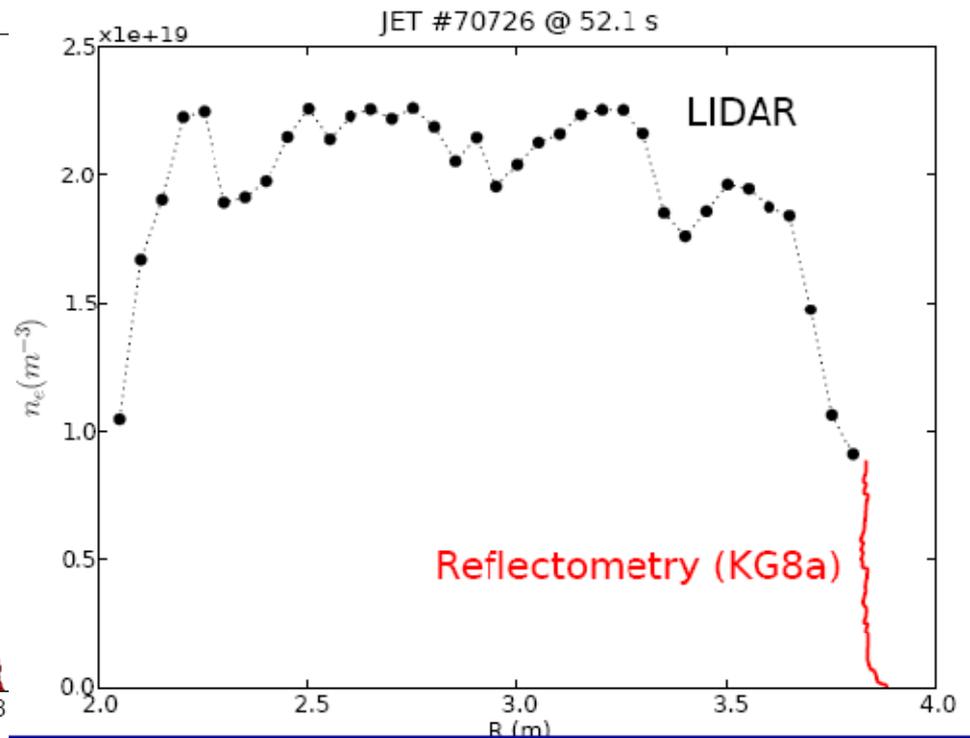


# JET – „sewing“ of density profile from the measurements of two different diagnostic method

(LIDAR - core and fast swept XM reflectometer - edge)



L mode



H mode

# Broadband reflectometer for COMPASS

(under development with IST Lisboa)

1. Edge pedestal plasma measurements with spatial resolution  $\Delta r \leq 1\text{cm}$
2. Correlation properties of plasma turbulence with **frequency hopping** (low phase noise synthesizers)
3. Study of fast transient events with time resolution  $\sim$  tens of  $\mu\text{s}$

System will have four frequency bands, but together five reflectometers:

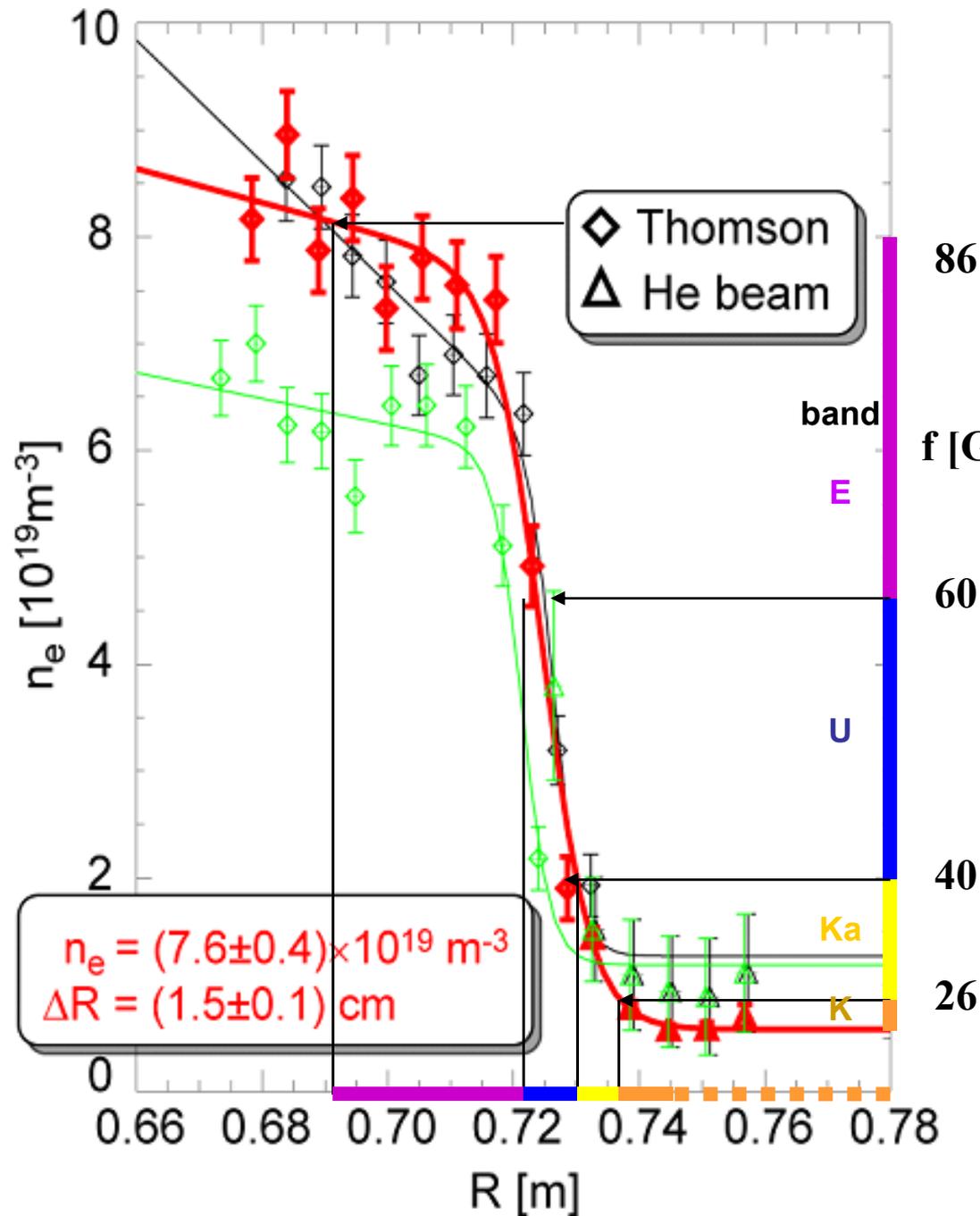
**Ka** in O- as well as X-mode,  
**K, U, E** only in O-mode

**COMPASS:**

**R = 0.56 m**

**a = 0.2 m**

$$N_{\text{crit}} [\text{m}^{-3}] = 0.0124 f^2 [\text{Hz}]$$



86

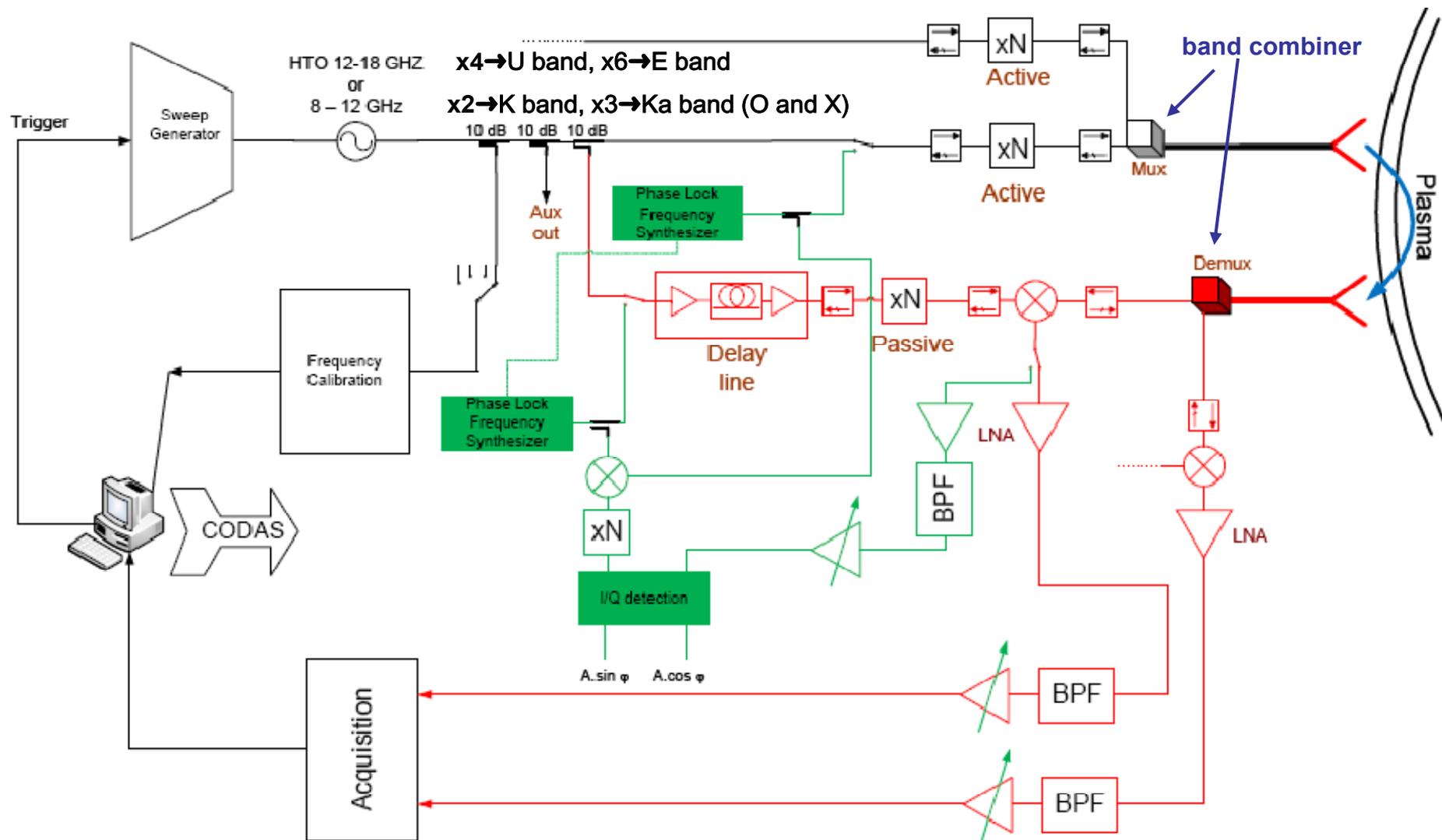
f [GHz]

60

40

26

# Block scheme of the transmitter and receiver of COMPASS reflectometer (design, under construction)



# Scattering of the electromagnetic waves

**Conservation laws:**  $\mathbf{k}_s = \mathbf{k}_i \pm \mathbf{k}_f$  where indices: **i ... incident radiation (wave)**  
 $\omega_s = \omega_i \pm \omega_f$  **s ... scattered radiation**  
**f ... scattering object**

**Angle of observation  $\Theta_s$  determines, what  $k_f$  is measured**

**For optical and microwave band in case of plasma turbulence detection may be seen that:**

$\omega_i \gg \omega_f \Rightarrow \omega_s \approx \omega_i$  and therefore also  $|k_i| \approx |k_s|$  ( $k = \omega/v$ )  $\Rightarrow$  vectors  $k_i$  a  $k_s$  form an isosceles triangle

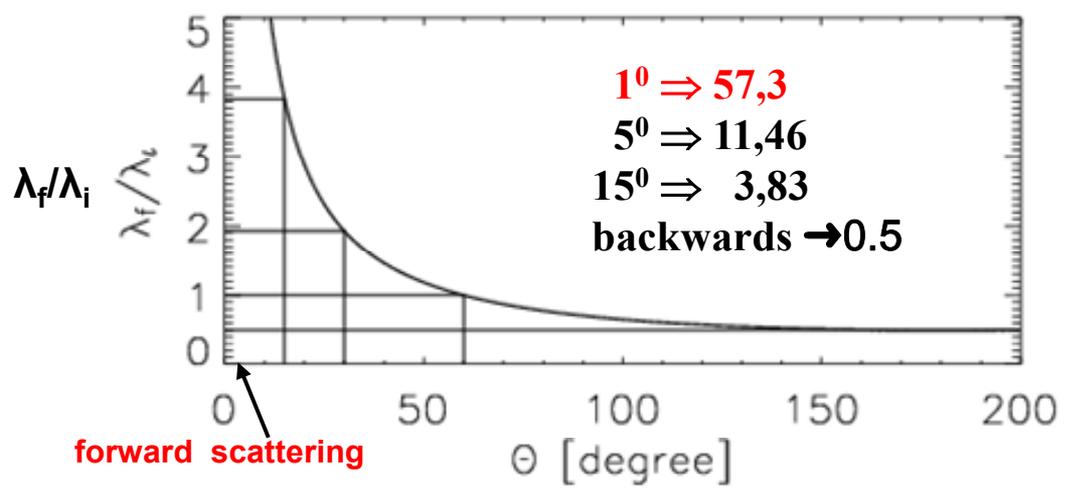
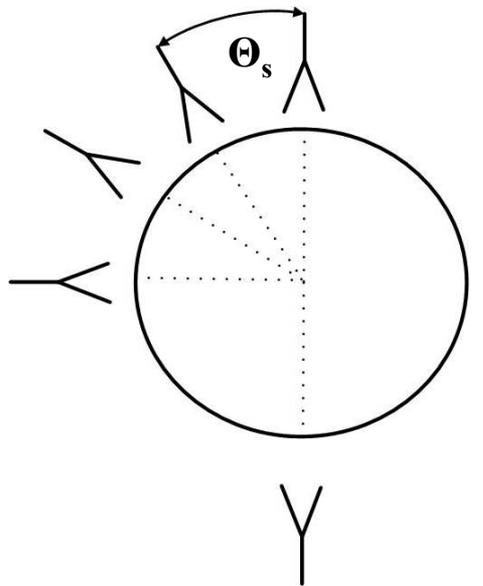
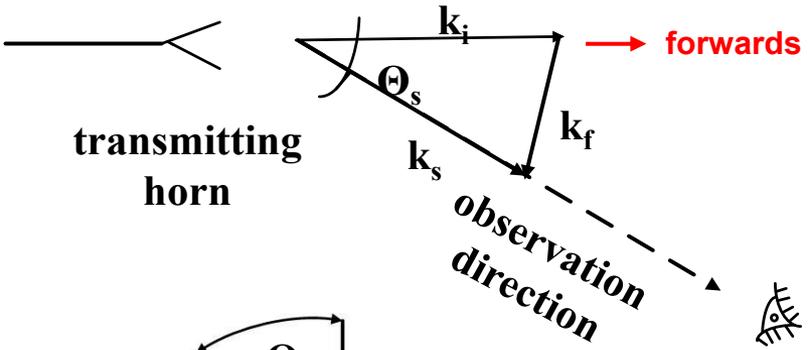
$$k_f^2 = k_i^2 + k_s^2 - 2k_i k_s \cos \Theta_s$$

$$\Rightarrow k_f = 2k_i \sin(\Theta_s/2) \quad (\lambda_f^{\min} = (\lambda_i/2))$$

$\Rightarrow$  angle of maximum scattering on inhomogeneity with a given  $\lambda_f$  fulfils the following condition

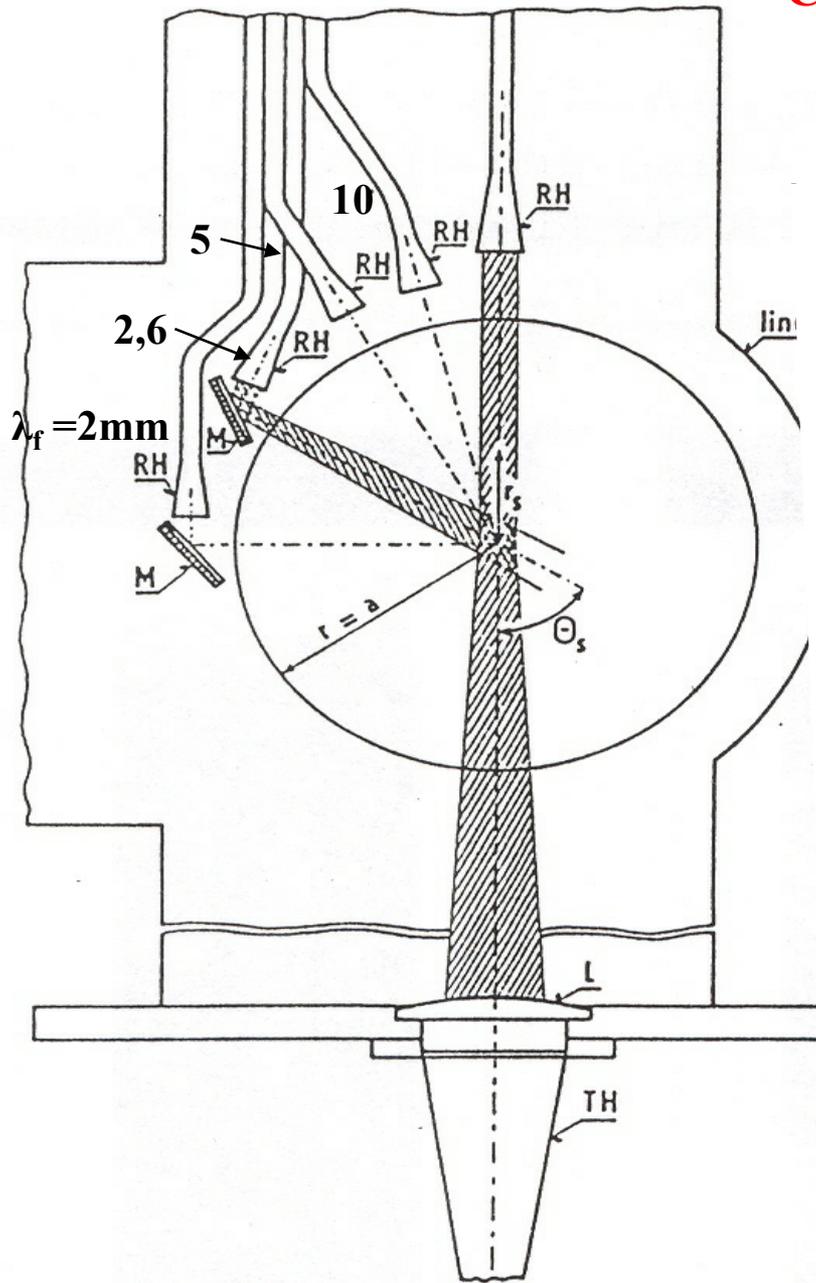
$$\Theta_s^{\max} = 2 \arcsin(\lambda_i / 2\lambda_f) \Rightarrow \lambda_i / \lambda_f = 2 \sin(\Theta_s / 2)$$

(so called Bragg condition, according to scattering of electrons or XR on the crystals: for  $\omega_i \gg \omega_f$  the most scattered wavelength equals the incident wavelength):

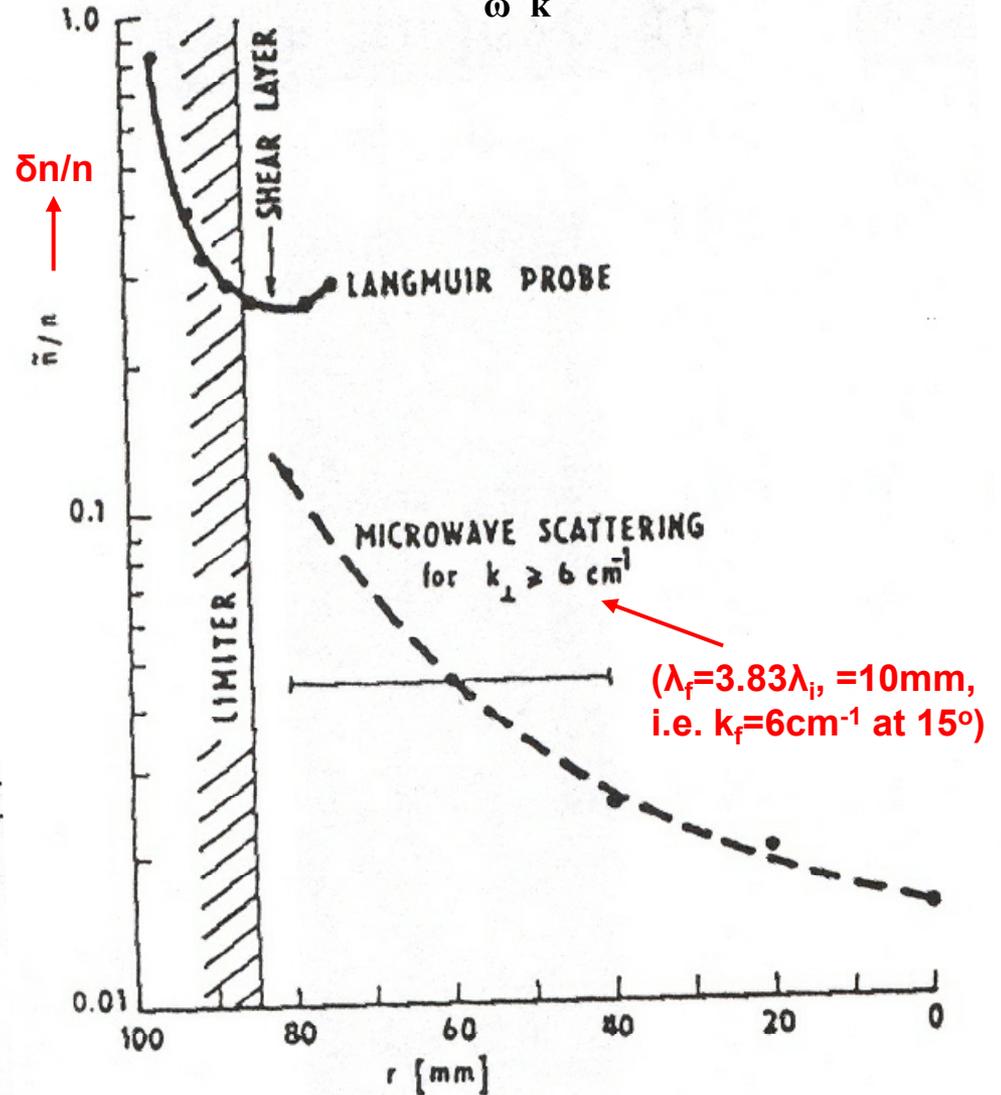


# CASTOR microwave scattering at $\lambda=2,6\text{mm}$

Fizika Plazmy 15 (1989), 515

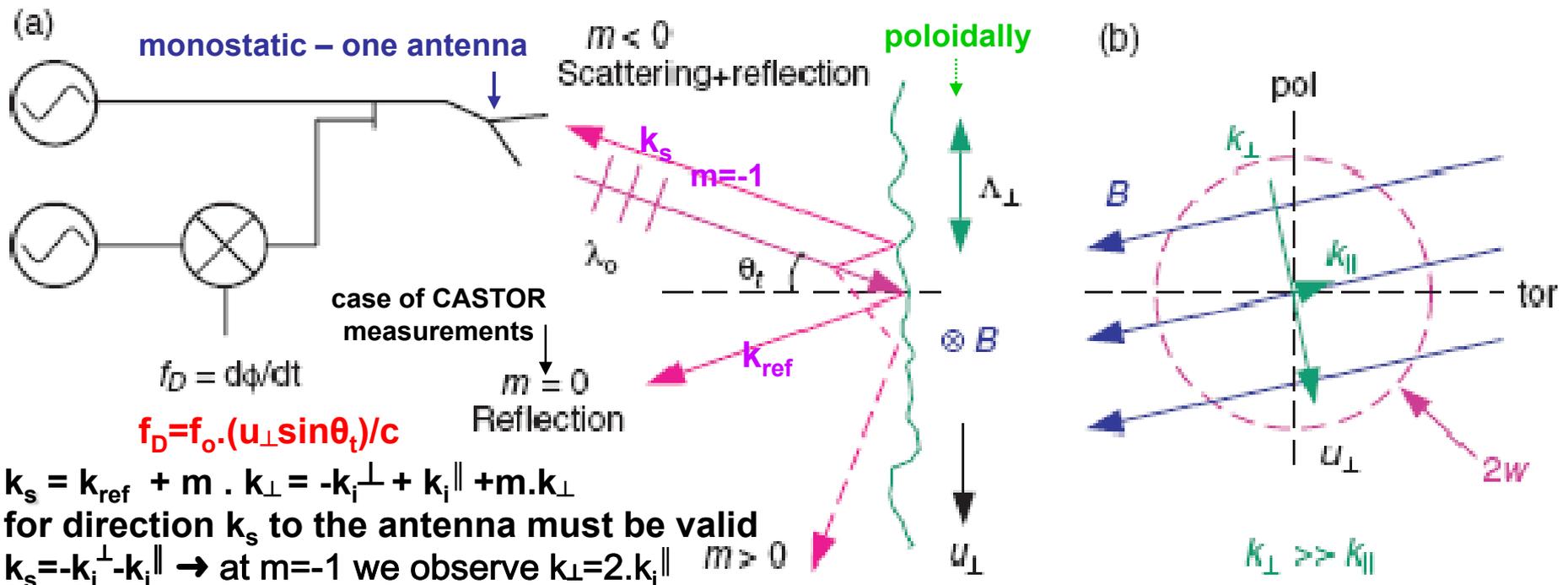


$$\langle \delta n^2 \rangle = n / (2\pi)^4 \cdot \int \int S(\mathbf{k}, \omega) \cdot d^3\mathbf{k} \cdot d\omega$$



# Scheme of monostatic Doppler reflectometry

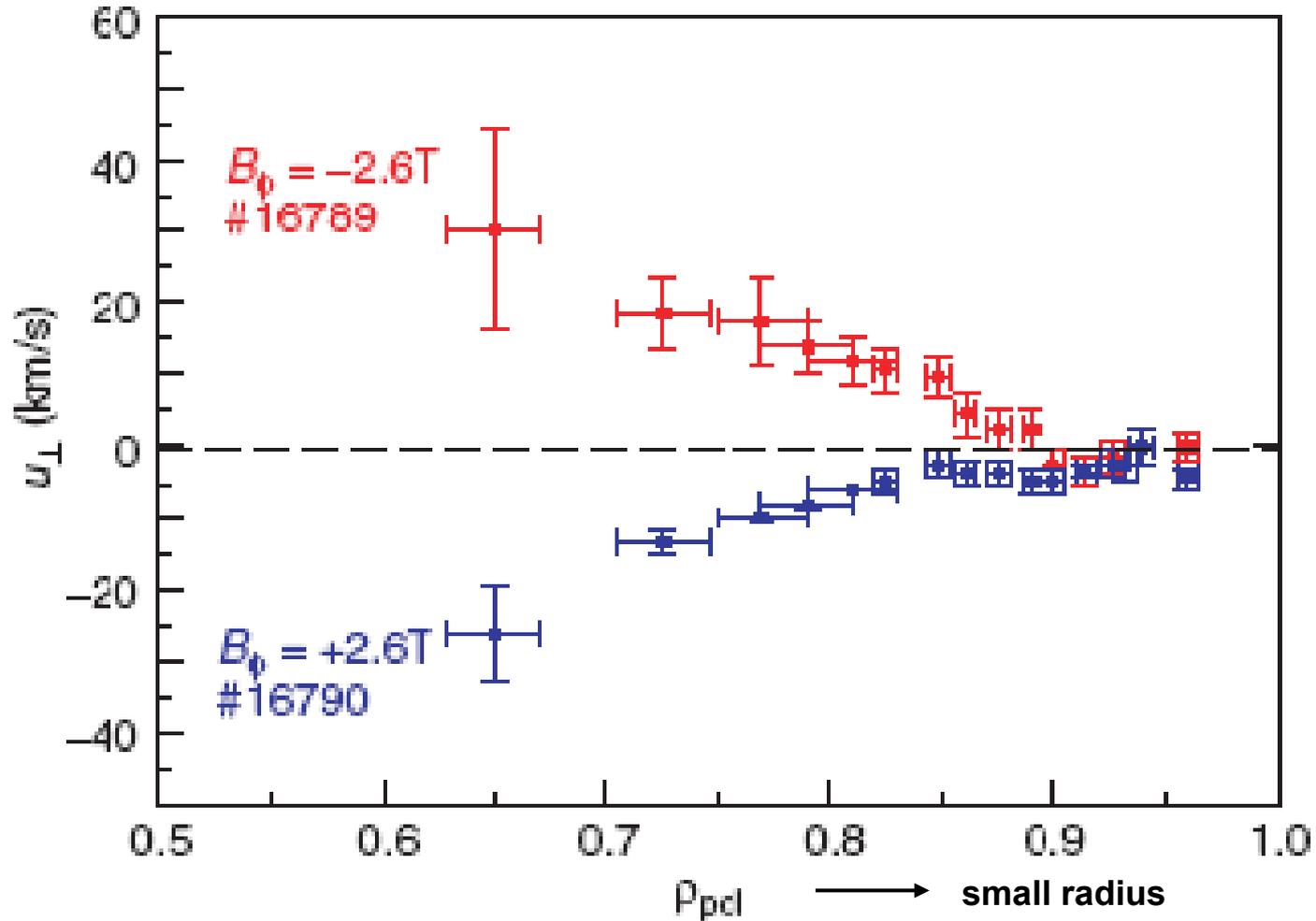
- a) microwave beam ( $\lambda_o, f=f_o$ ) strikes upon a fluctuating reflecting surface ( $\Lambda_{\perp}=2\pi/k_{\perp}$ , amplitude  $\sigma$ ), moving across the magnetic field with a velocity  $u_{\perp}$  ( $k_{\parallel}\approx 0$ ), under angle  $\Theta_t$  (measured from the normal of incidence)
- b) angle  $\Theta_t$  determines the measured  $k_{\perp}$  fluctuations (Braggs condition),  $f_D = \delta f_o$  determines  $u_{\perp}$ , intensity of the reflected signal determines the amplitude of fluctuations  $\sigma$
- b) if dimension of the reflecting „spot“ is  $2w$  and wave strikes under angle  $\Theta_t$  to the magnetic field lines then the space resolution of  $k_{\perp}$  measurement  $\Delta k_{\perp} \approx 2 \cdot \sqrt{2} \cdot \cos\Theta_t / w$  and  $\delta\omega_D \approx \Delta k_{\perp} \cdot u_{\perp}$



# Tokamak AUG – two profiles of $u_{\perp}$ for different directions of the magnetic field (O-mode reflectometer)

measured at 7,5MW co-NBI,  $I_p = +0.8\text{MA}$ , just after L $\rightarrow$ H transition

$B_{\text{tor}} = -2,6\text{T}$  (red),  $B_{\text{tor}} = +2,6\text{T}$  (blue)



# Comparison of microwave reflectometry and interferometry

## A) Common features of the both methods:

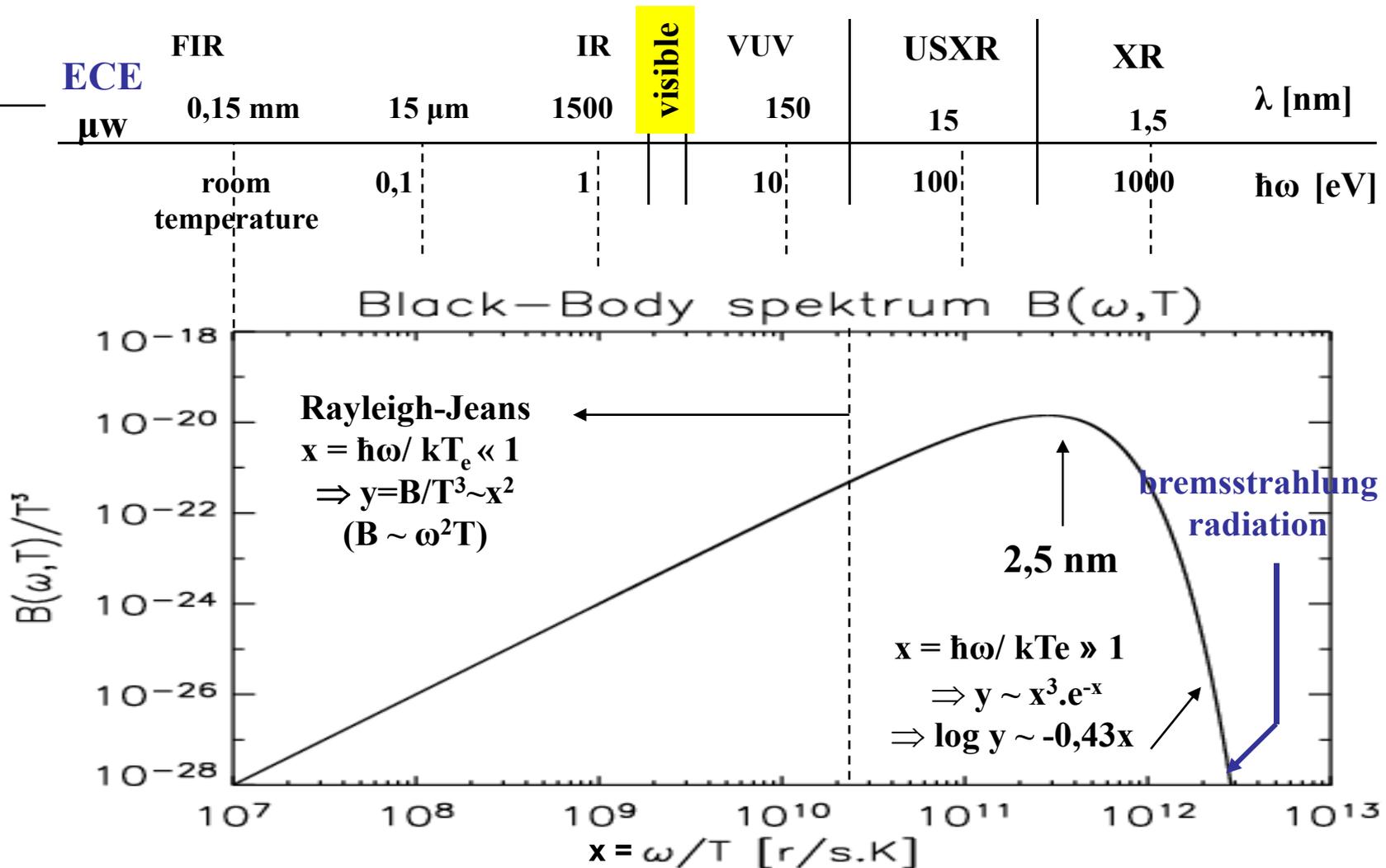
- in both cases **phase interferometric measurements** of the probe electromagnetic wave is carried out
- i.e. the both methods are **active** diagnostic methods
- the probing is realized in **contactless** way, i.e. both methods are convenient for diagnostics of a hot plasma
- **the same RF technique** (waveguides, antennas, ...) is used
- **the same RF generators** (frequencies may differ, but maximum several times)
- **the same phase detecting systems** (interference of the probing and reference waves) are used

# Comparison of microwave **interferometry** and **reflectometry**

B) The methods have **different characteristics** and this fact results in a **different application** of the both methods:

	<b>interferometer</b>	versus	<b>reflectometer</b>
• probing wave	passes the medium ↓		is reflected on the cut-off layer $r_{\text{cut}}$ where $r_{\text{cut}} = \text{function}(\omega, n_e, B)$ ↓
• effect of the the medium is	averaged along the wave path ↓		extremely well localized ↓
• utilization	determination of $\langle n_e \rangle_{\text{lin}}$		determination of $\delta n_e(r=r_{\text{cut}})$
• sensitivity to $\delta n_e$	low		high
• data evaluation	simple (low f, small amplitude changes) ↓		more complex (high f, high amplitude changes) ↓
• measuring circuit	interferometer with one phase detector		two interferometric branches are needed (I/Q detection principle, on CASTOR sin-cosin scheme)

**Planck:  $B(\omega, T) = (\hbar/8\pi^3c^2)\omega^3 \cdot (1/(e^{\hbar\omega/kT_e} - 1))$**   
**for CASTOR, where  $T_e=100\text{eV}$ :**



# Tokamak DITE – evaluated X-wave ECE spectrum (and in the case of maxwellian plasma electron temperature profile)

for following parameters on the device axis:

- density  $6.19^{19} \text{m}^{-3}$
- electron temperature 300 eV
- magnetic field 2,7 T

Tokamaks:

$\omega_c(R) = eB(R)/m \rightarrow$  radius identification

spectral x space resolution:  $d\omega/\omega = -dR/R$

1. Tokamaks- spectral x space resolution:

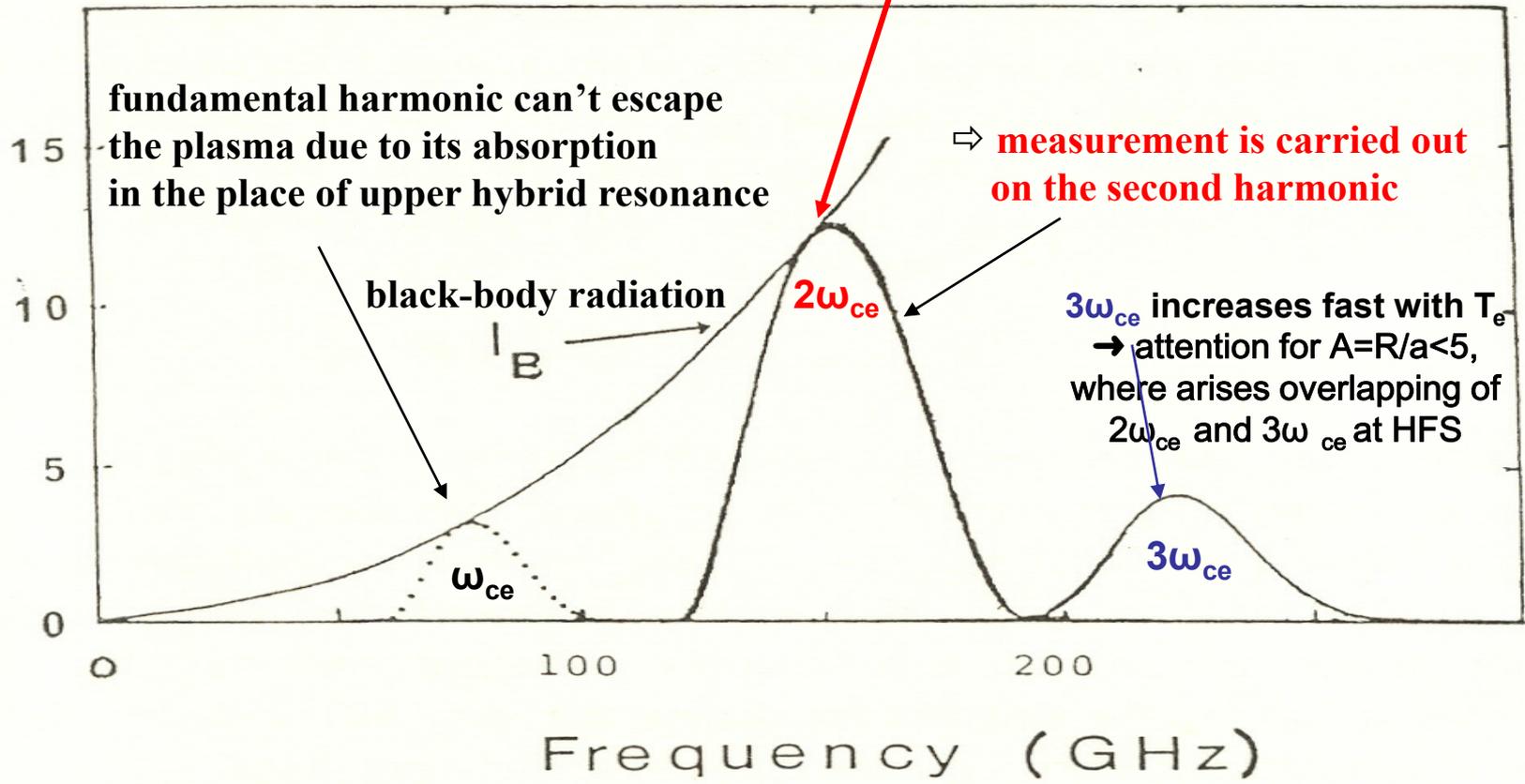
$$d\omega/\omega_{ce} = s \cdot (v_{Te}/c)^2 = -dR/R$$

2. Optical thickness must be:  $\tau = \int_{s=1}^2 \alpha_\omega(N, T_e, R, s) \cdot dR \gg 1$

$$I_{pl} = I_B \sim T_e$$

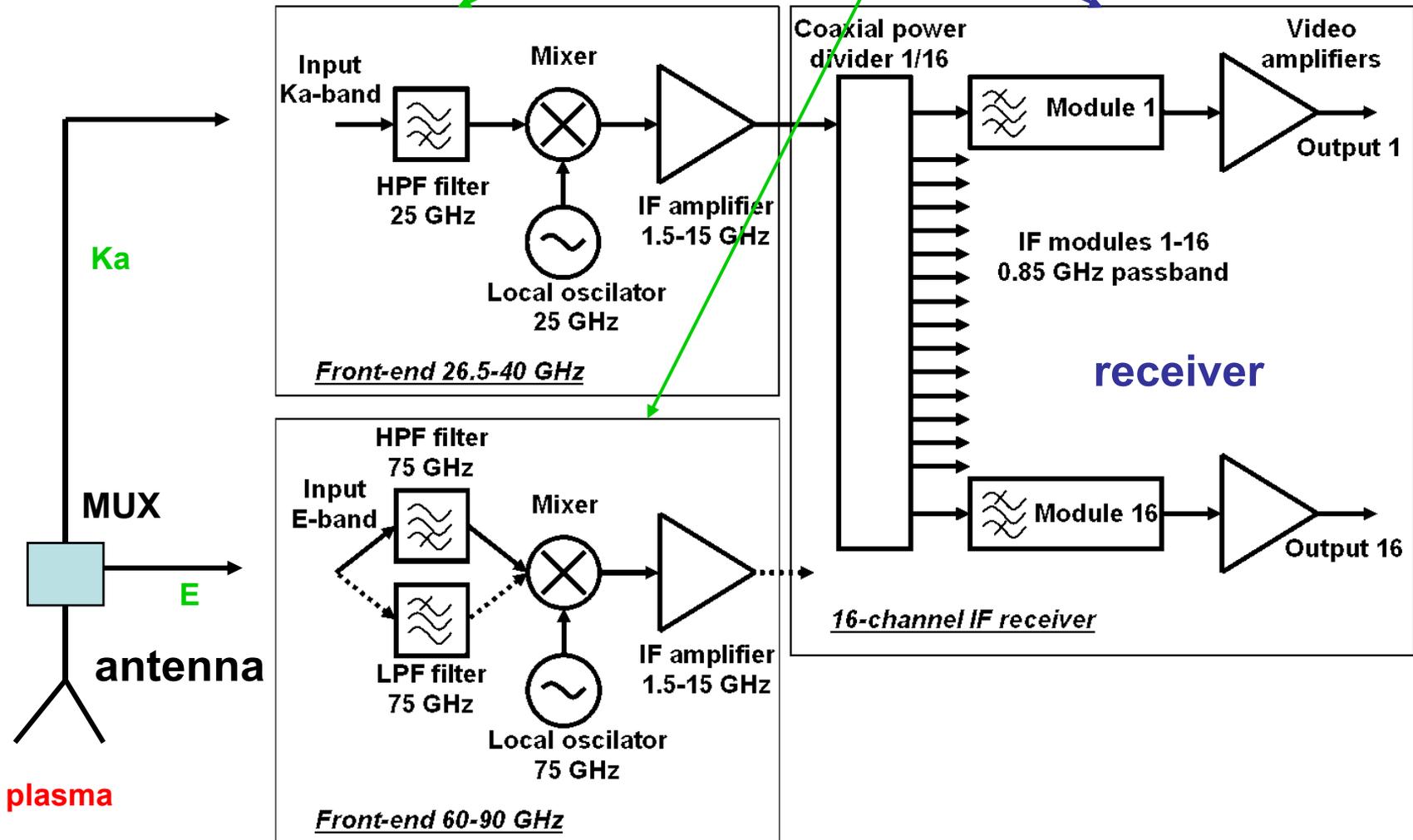
$\omega/\omega_{ceo}$

Intensity ( $\text{pW Hz}^{-1} \text{m}^{-2} \text{sr}^{-1}$ )

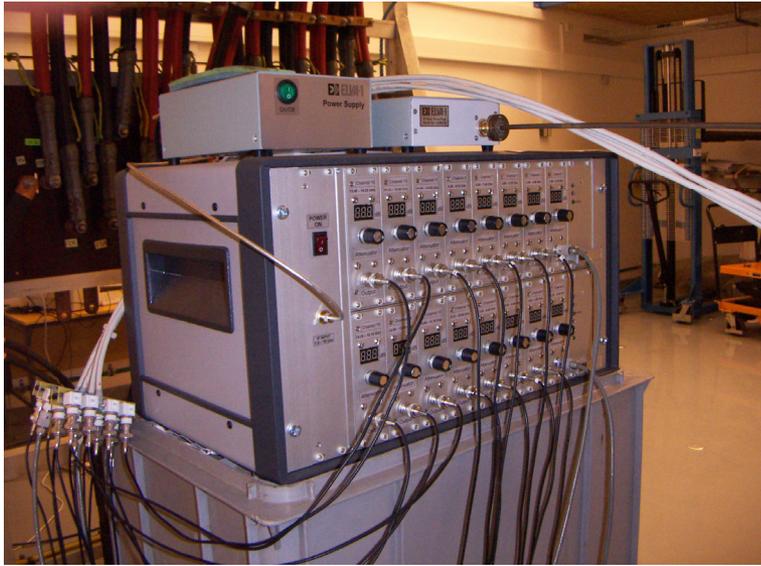


# Radiometer for COMPASS: ECE ( $\theta=90^\circ$ ) and EBE ( $\theta\neq 90^\circ$ ) both polarizations

One 16-channel heterodyne receiver with total bandwidth 15GHz will be alternately connected to Ka-band or E-band front-end circuits (E-band is divided to two sub-bands using a changeable input BPF)



# Equipment



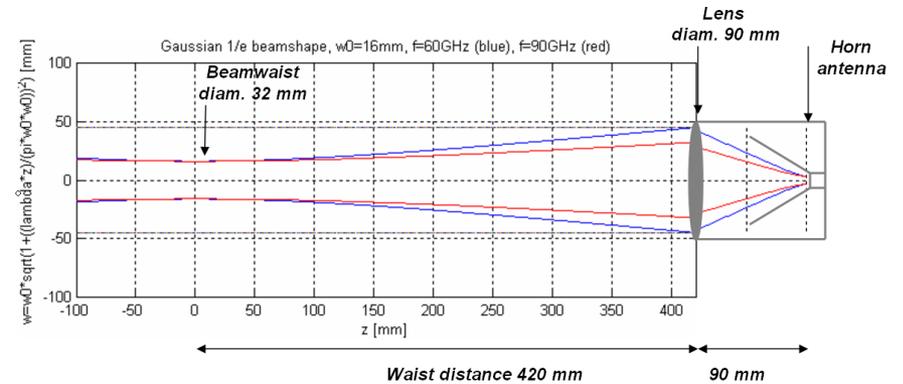
16-channel receiver



GOLA antennas for Ka and E bands

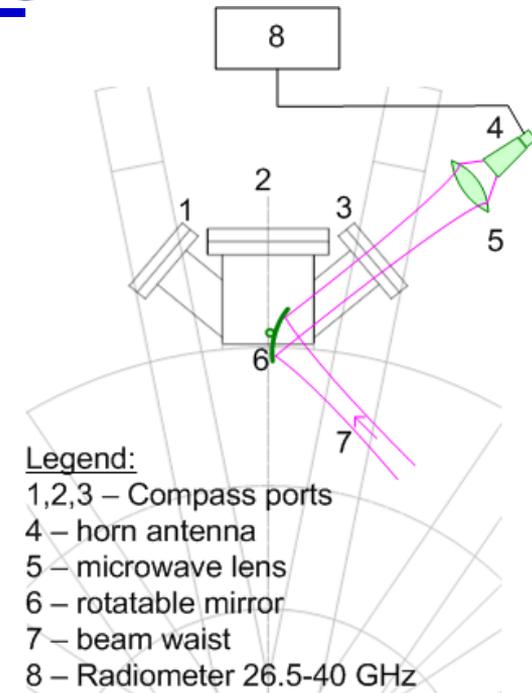
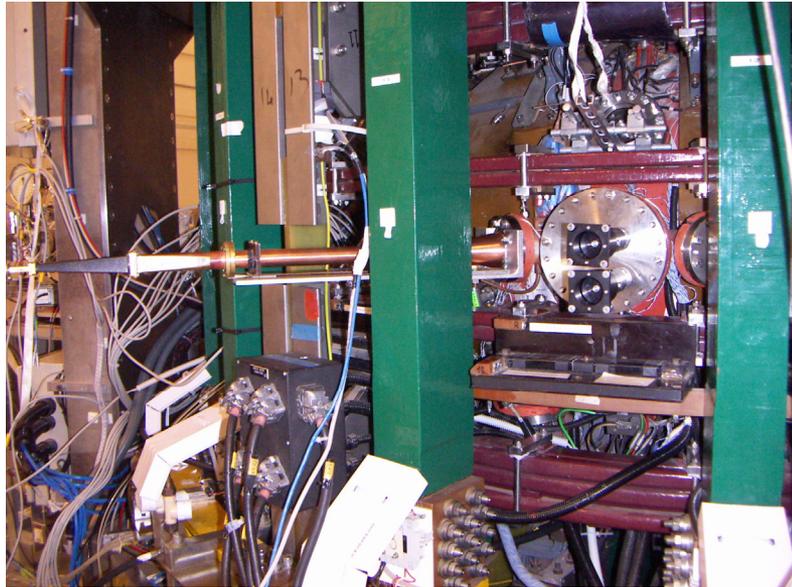


Horn antenna



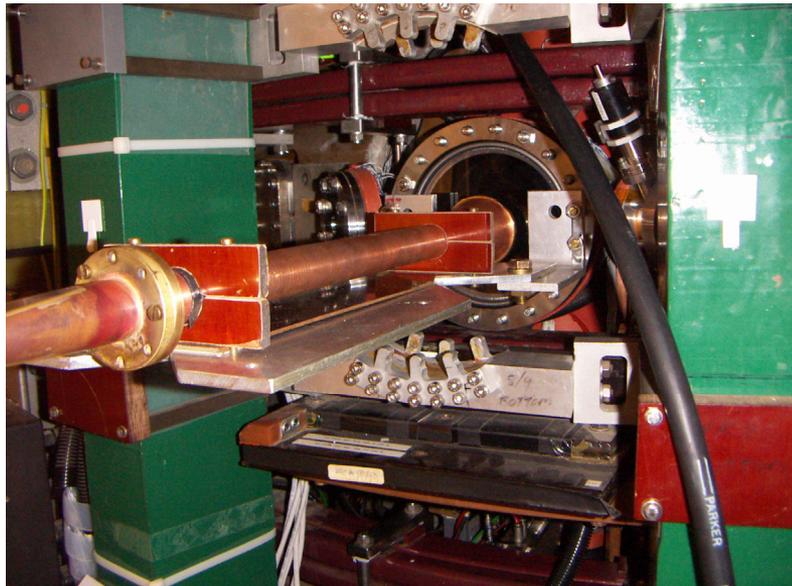
Beamshapes of gaussian-optics lens antenna

# First tests on Compass



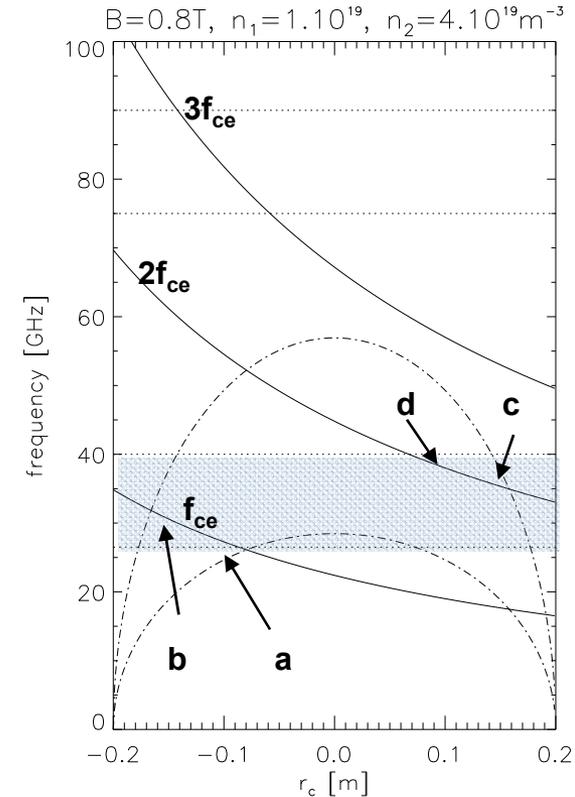
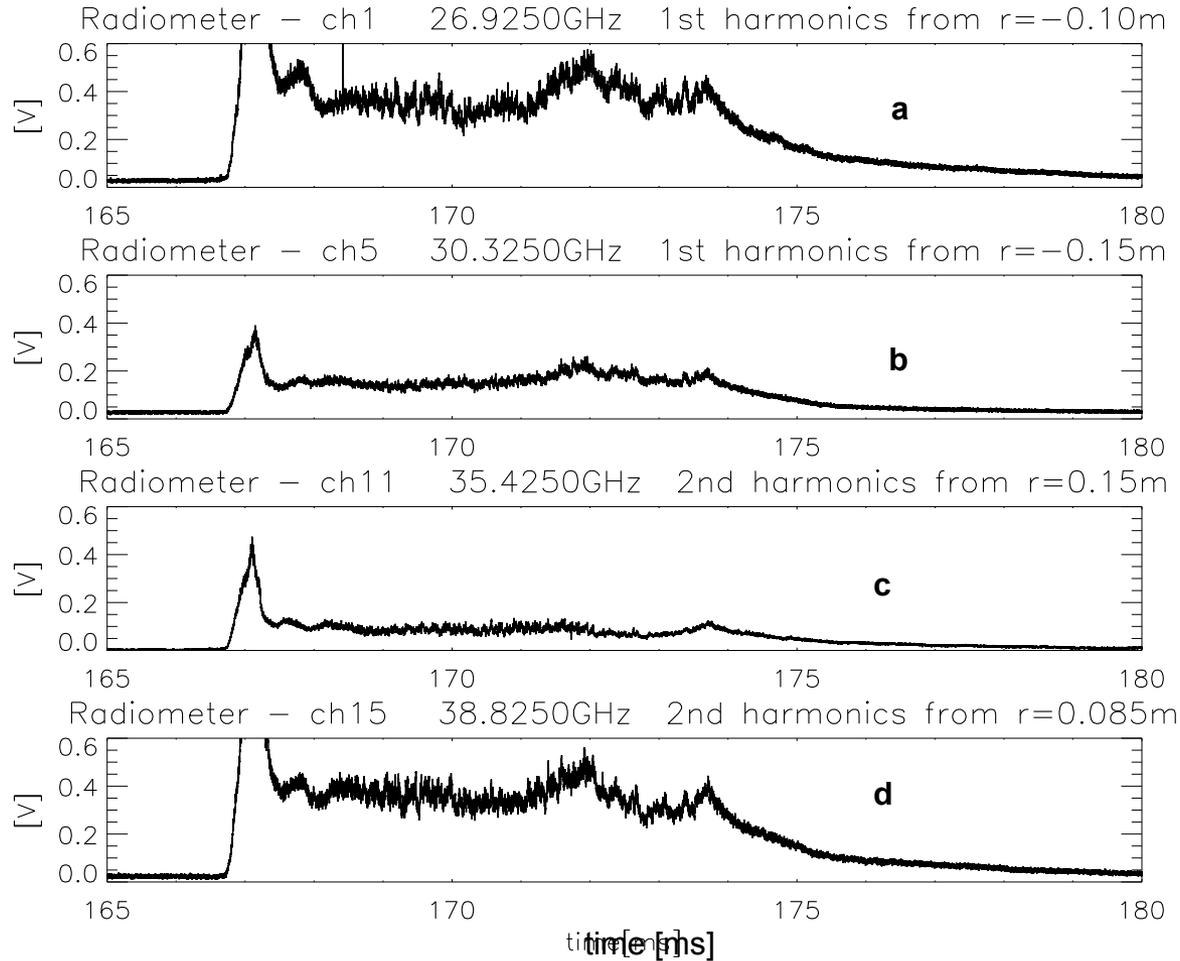
Legend:

- 1,2,3 – Compass ports
- 4 – horn antenna
- 5 – microwave lens
- 6 – rotatable mirror
- 7 – beam waist
- 8 – Radiometer 26.5-40 GHz



# Example of signals from tests on COMPASS

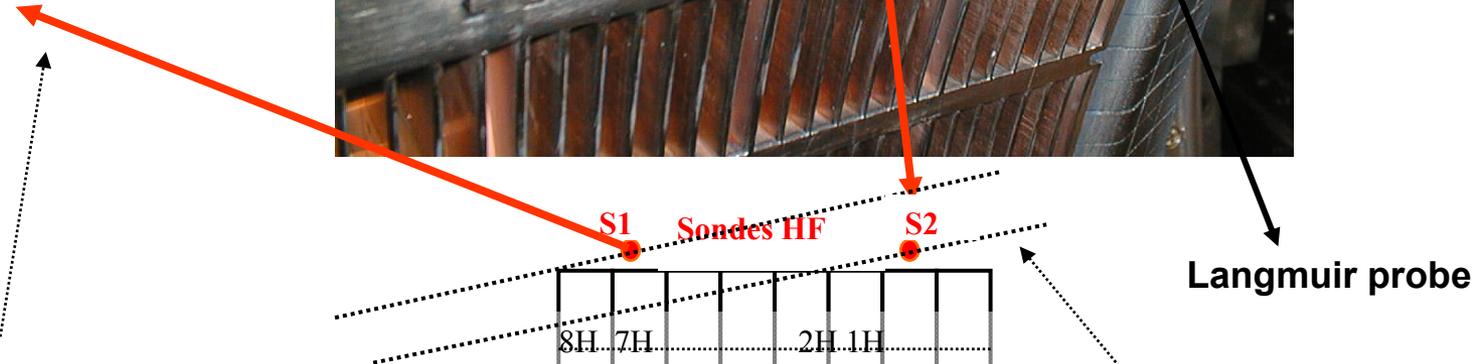
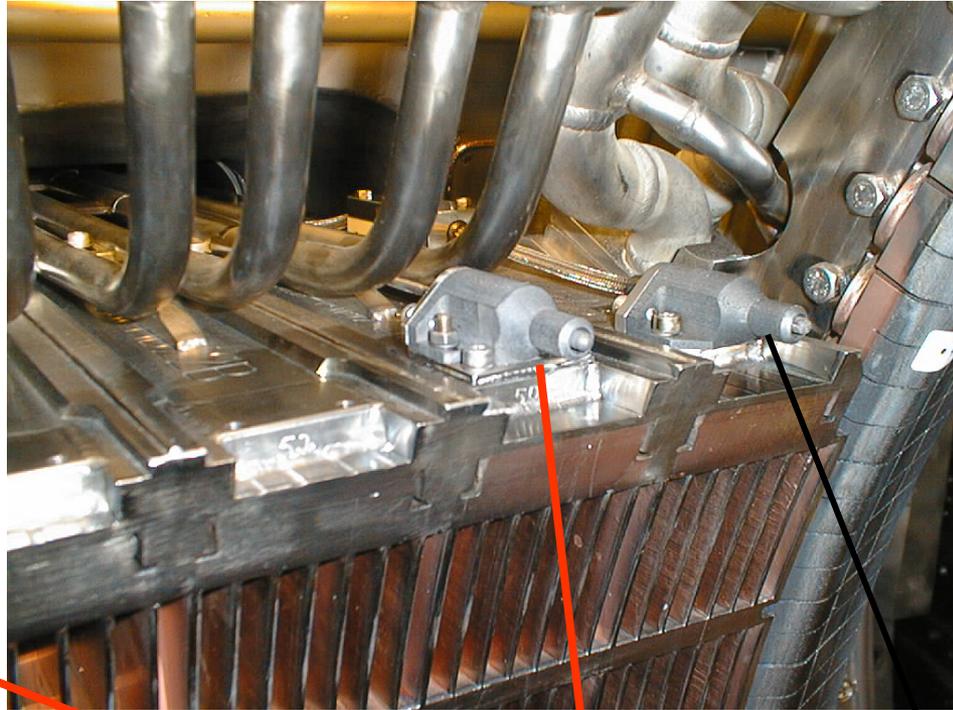
( $B_0 = 0.8$  T, Ka band, O-mode)



0.8 T / Ka band

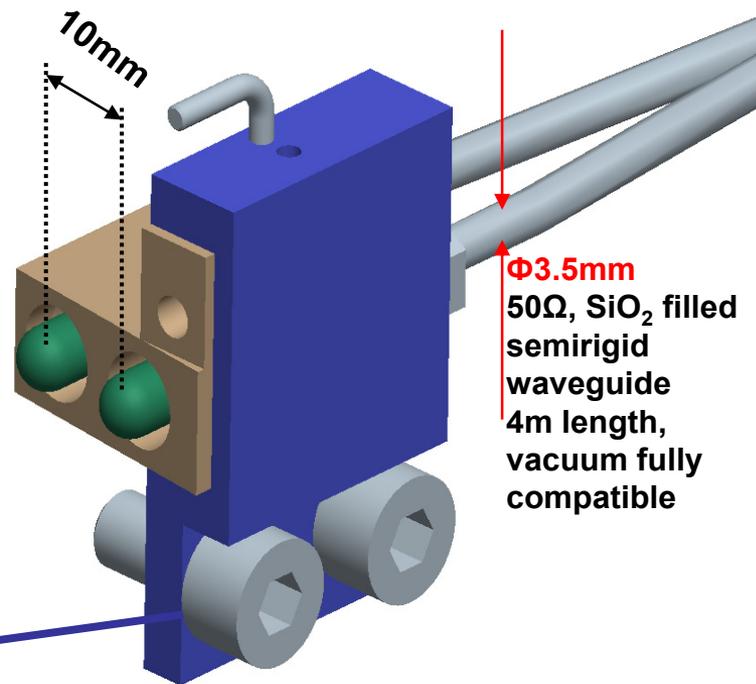
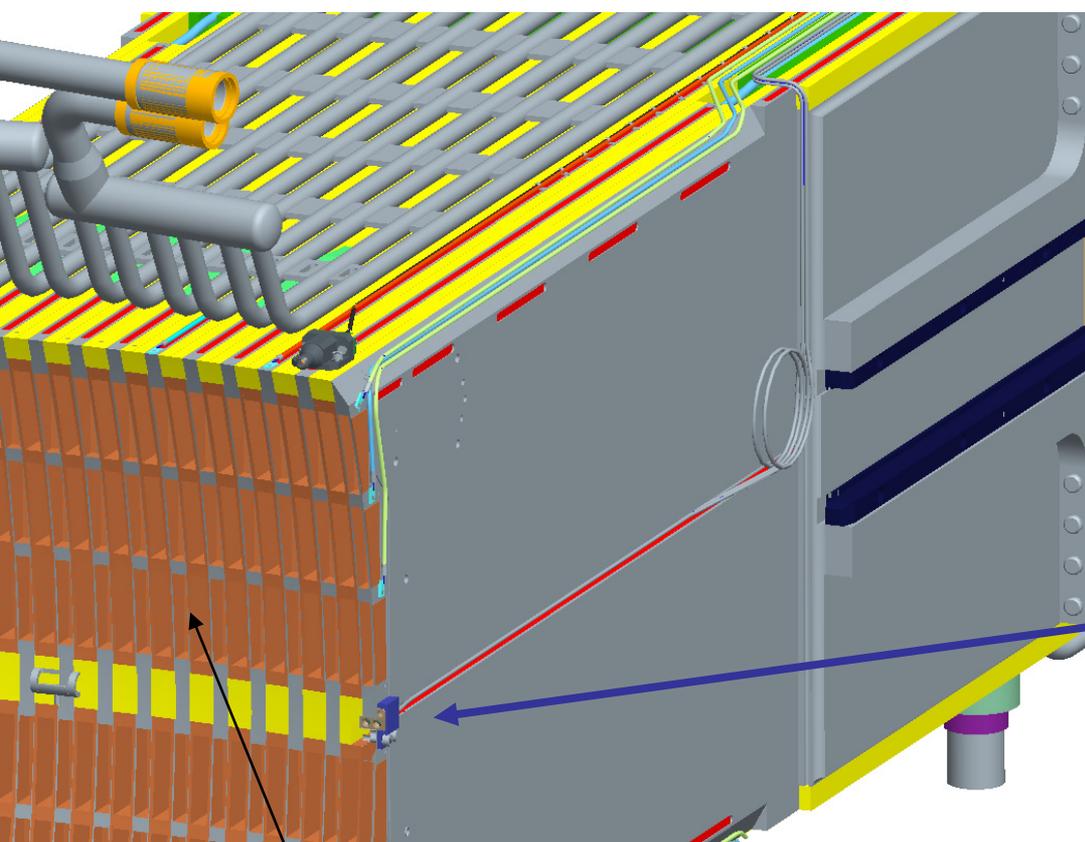
COMPASS:  $A=R/a = 0.56/0.2=2.8$

# Diagnostic de sondes HF sur le coupleur C3



Two RF 50 $\Omega$  probes:  
- additionally mounted  
- the same outer form as the Langmuir probes there already located  
(vacuum, TS long-pulse compatible)

**Miniature double 50Ω RF probe for new C4 Tore Supra LHCD launcher**  
(correlation characteristics of the launched wave as well as production of parasitically accelerated electrons resulting in creation of hot spots observed under high RF powers)



**RF probe (RF “sniffer”)**  
(toroidally curved, radially movable,  
expected temperature more than 1000°C)

**C4 launcher 3.7GHz**

Thanks for your attention