CONFINEMENT I

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CONTENT

Introduction I

To achieve the fusion reaction in plasmas, some kind of the confinement is necessary. Depending on plasma density, two kind of confinement are considered

a) inertial confinement

b) magnetic confinement

Introduction II - Inertial fusion

The characteristic of inertial confinement is that the extremely high-density plasma is produced within a short period by means of an intense energy driver, such as a laser or particle beam, so that fusion reaction can occur before the plasma starts to expand. Here, therefore, the problem of confinement of plasma is avoided.

Introduction III - Magnetic Confinement I

- For a plasma with density of 10^{20} m^{-3} particles, the magnetic confinement can be used.
- Magnetic confinement systems are broadly classified as open systems and closed systems.
  - In open systems some magnetic field lines can leave the system,
  - in closed system all magnetic field lines stay within a certain region.
- An example of the open system is so called “magnetic mirror“. “Tokamak“ is a typical closed system.

Introduction III - Magnetic confinement II

- To understand to the mechanism of magnetic confinement in both systems, the knowledge of the dynamics of single charged particles in magnetic and electric fields is necessary.
- The confinement can be realized in
  - magnetic mirror systems, using the constancy of the magnetic moment, or in
  - toroidal systems, using the possibility of averaging of curvature drift.

Mirror confinement I

Let us consider a magnetic field whose magnitude varies in „z“ direction. Let the field be axisymmetric with

$$B_\theta = 0, \frac{\partial}{\partial \theta} = 0$$

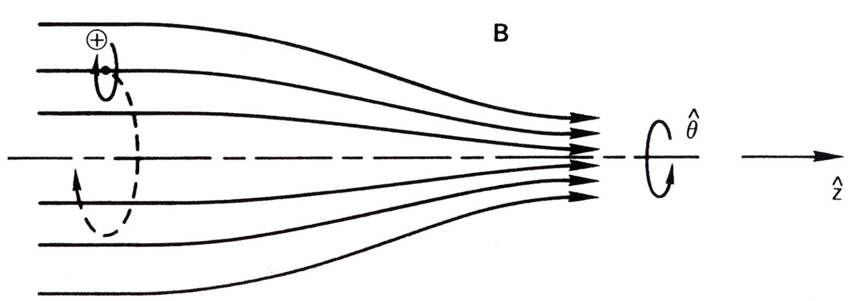
Considering slow changes of the magnetic field, it is possible to prove constancy of the magnetic moment defined as

$$\mu = \frac{1}{2} \frac{m v_\perp^{\ 2}}{B}$$

where $v_\perp$ is the perpendicular component of the velocity ***v***.

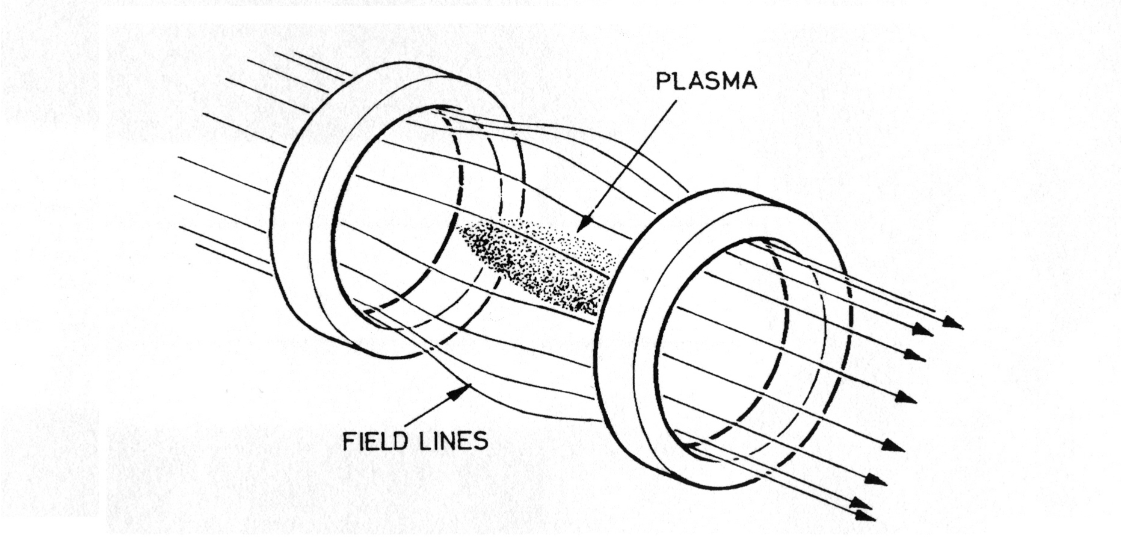
Mirror confinement II

- Since the energy must be conserved, the increase of the perpendicular energy must be at the expense of the parallel energy.
- Thus it may happen that for some value of ***B*** the parallel energy decreases to zero value, and particle is reflected.



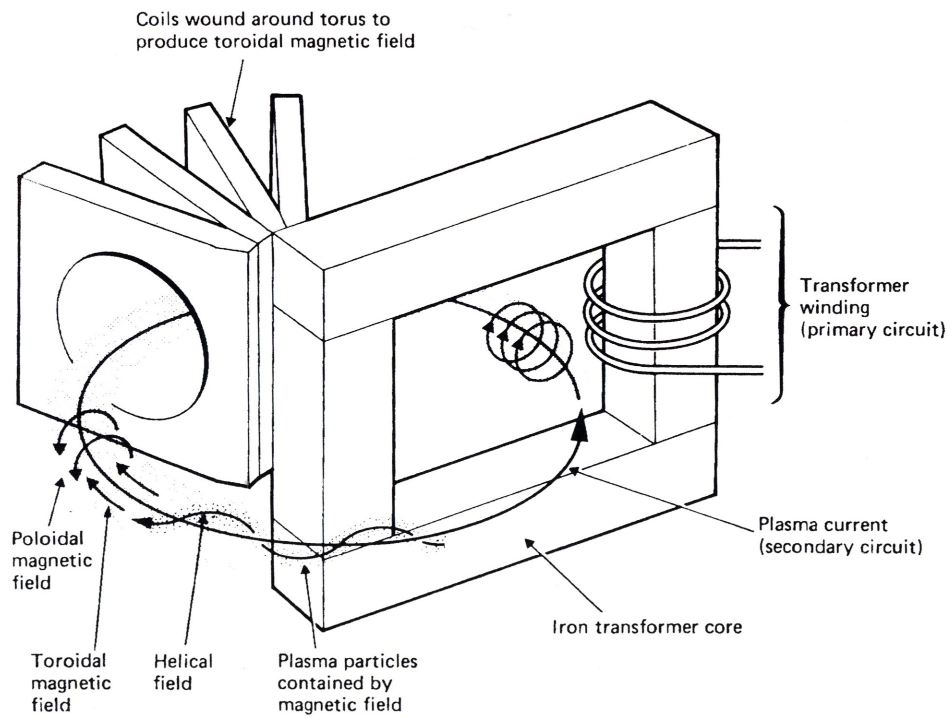
Mirror confinement III - Realization of mirror confinement

Simple mirror confinement : magnetic field is formed by two Helmholtz coils.



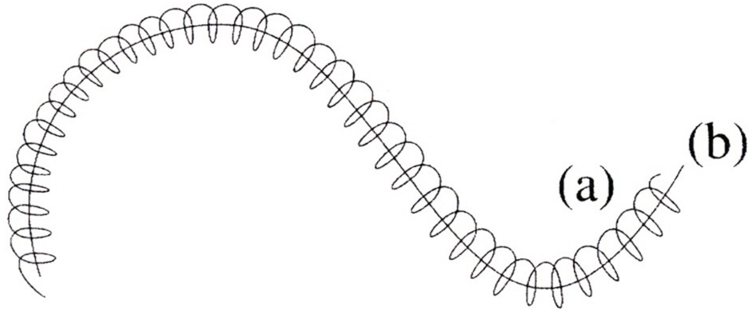
Tokamak confinement I

System of magnetic field lines



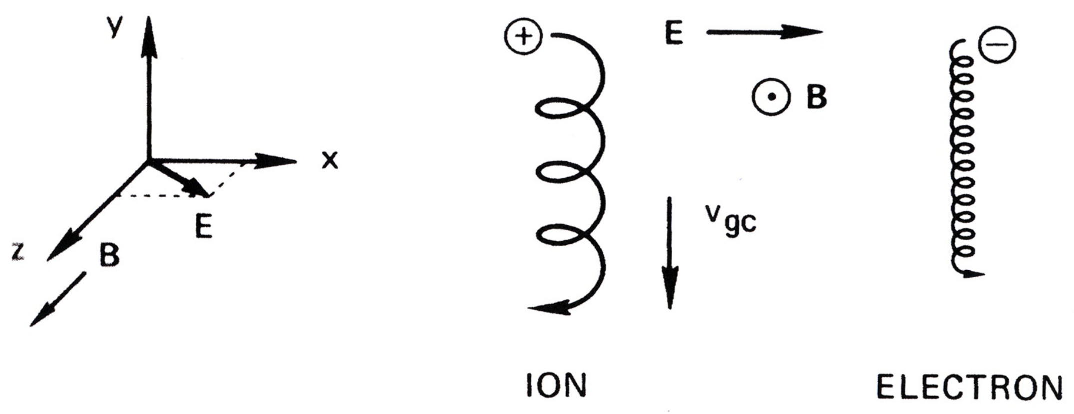
Tokamak confinement II - Drift trajectories

The position of the centre about which the particle gyrates is called guiding centre. Usually, in the description of particle movement, the trajectory, formed by the guiding centre is followed. This trajectory is called drift trajectory (b).



Tokamak confinement III - Drifts in crossed *ExB* fields

Drift velocity v_E : $$\mathbf{V}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$



Tokamak confinement IV - Drift velocity $\mathbf{v}_f$ in the field of a general force **F**

The foregoing result can be applied to other forces by replacing q**E** (q is the charge) by a general force **F**

$$\mathbf{v}_f = \frac{1}{q}\frac{\mathbf{F}\times\mathbf{B}}{B^2}$$

This force can be e.g. the centrifugal force

$$\mathbf{F}_{cf} = \frac{m v_{//}{}^2}{R_c}\hat{\mathbf{r}} = m v_{//}{}^2 \frac{\mathbf{R}_c}{R_c{}^2}$$

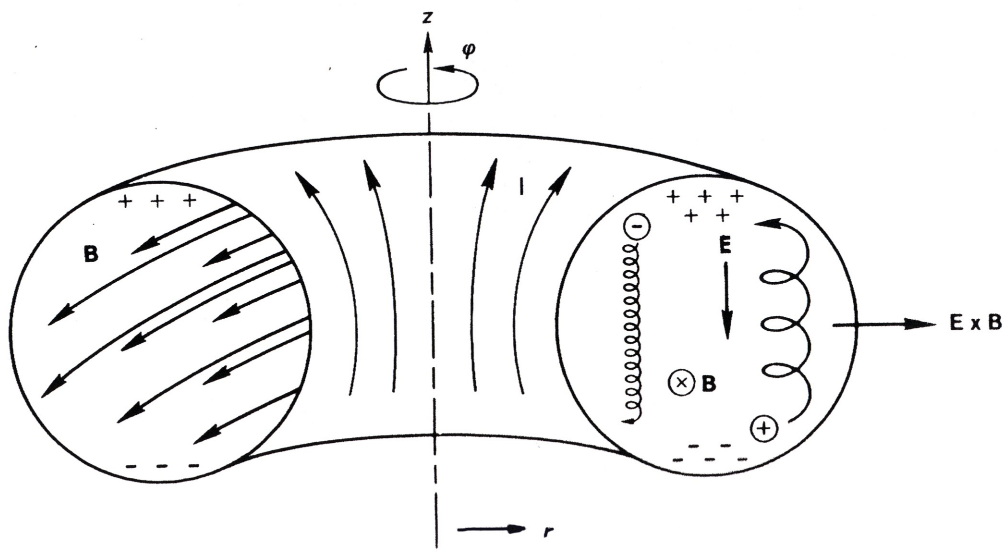
Tokamak confinement IV - Curved magnetic field lines. Curvature drifts

Drift in the magnetic field with curved magnetic field lines with the radius of the curvature $\mathbf{R}_C$

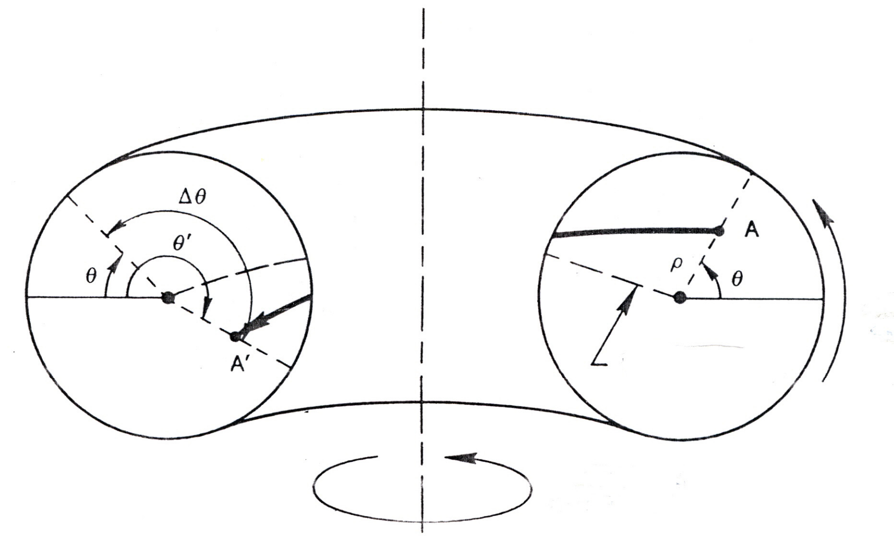
is the drift velocity **v**:

$$\mathbf{V} = \frac{m}{q}\left(v_{//}{}^{2} + \frac{1}{2} v_{\perp}{}^{2} \right) \frac{\mathbf{R}_c \times \mathbf{B}}{R_c{}^{2} B^{2}}$$

Tokamak confinement V - Drift in toroid: magnetic field lines are circles



Tokamak confinement IV- Drift in toroid: helical magnetic field lines



Roughly, the helical field lines move upward and downward the equatorial plane and in such a way, the curvature drift of particles is compensated.

Collisions as a starting mechanism of plasma transport

- Single particle confinement in tokamaks (i.e. confinement, where particles are subjected only to external magnetic field) offers confinement in a certain closed volume. Considering realistic conditions, even in a fully ionised plasma, Coulomb collisions represent one from the channels of escaping of particles.
- Nevertheless, it was experimentally found that e.g. thermal electron transport can be up to two orders higher than that predicted one from collisions.
- To explain this so called anomalous transport, which is obviously connected with different forms of MHD instabilities, is the most important challenge of tokamak plasma theory. Unfortunately, at the time, there is no convincing theoretical model.
- It is inevitable, however, that the route to understanding of this complicated problem must start with an analysis of the transport arising from collisions.

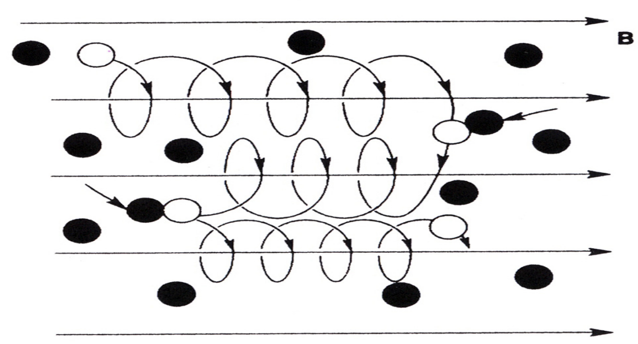
Important steps of collisional tokamak plasma theory

In what follows we shall discuss the following parts of collisional plasma theory (and at the same time the historical route to the understanding of tokamak plasma transport):

- Resistive plasma diffusion
- Pfirsch-Schlüter diffusion
- Banana regime transport
- Plateau transport
- Bootstrap current

Resistive plasma diffusion I

Resistive plasma diffusion is usually derived from two fluids equation: the generalized Ohm´s law and the pressure balance equation. The same results can be obtained from the discussion of diffusion of particles in a cylindrical plasma, immersed in an external magnetic field. The problem is well illustrated in the following picture.



White particle collides with black ones and jumps across magnetic field lines.

Resistive plasma diffusion II

In this case, the collisional transport can be expressed in terms of a simple diffusion process, called classical diffusion. The particles suffer collisions with a characteristic collision time, τ_c

A collision allows the particles to step across the magnetic field lines with a step length equal to the Larmor radius ρ_c

For electrons, this radius is given as $\rho_c = \sqrt{2} m_e v_{Te} / eB$

and the collision time is $\tau_e = 3(2\pi)^{3/2} \dfrac{\varepsilon_0^{\,2} m_e^{\,1/2} T_e^{\,3/2}}{n_i Z^2 e^4 \ln \Lambda}$

The diffusion coefficient is given as $D = \dfrac{\rho_c^{\,2}}{\tau_e}$

Here, v_{Te}, $\ln\Lambda$ are electron thermal velocity and the Coulomb logarithm.

Pfirsch-Schlüter diffusion

- This sort of diffusion generalizes the resistive diffusion, originally developed for cylindrical system, for the toroidal system. The toroidicity, namely, brings new phenomena.
- The reason for that is that pressure of plasma generates a force, directed outward along the major axis. Due to that, an electric field and current appear. Little bit complicated algebra gives the resulting diffusion in the form

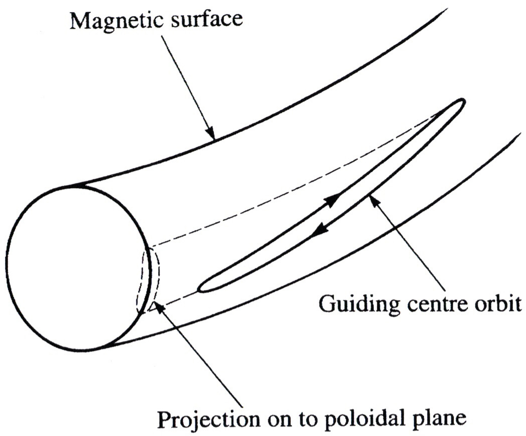
$$D = D_c \left(1 + 2 \frac{\eta_{//}}{\eta_{\perp}} q^2 \right)$$

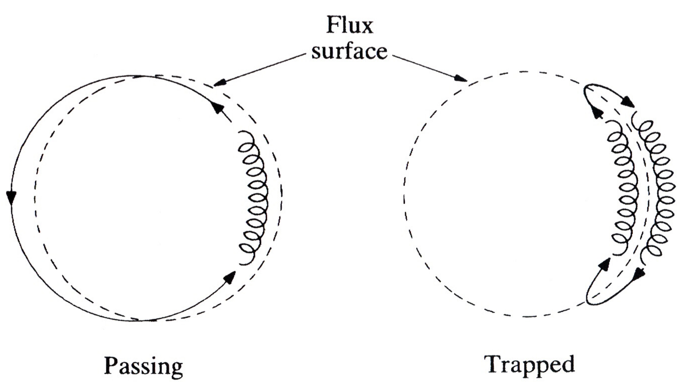
- Here, $D_c, \eta_{//}, \eta_{\perp}$ is classical diffusion coefficient, and parallel and perpendicular resistivity; q is safety factor.
- In this regime, sufficient collisionality is assumed. There, no trapping of particles (e.g. in bananas, see later) exists.

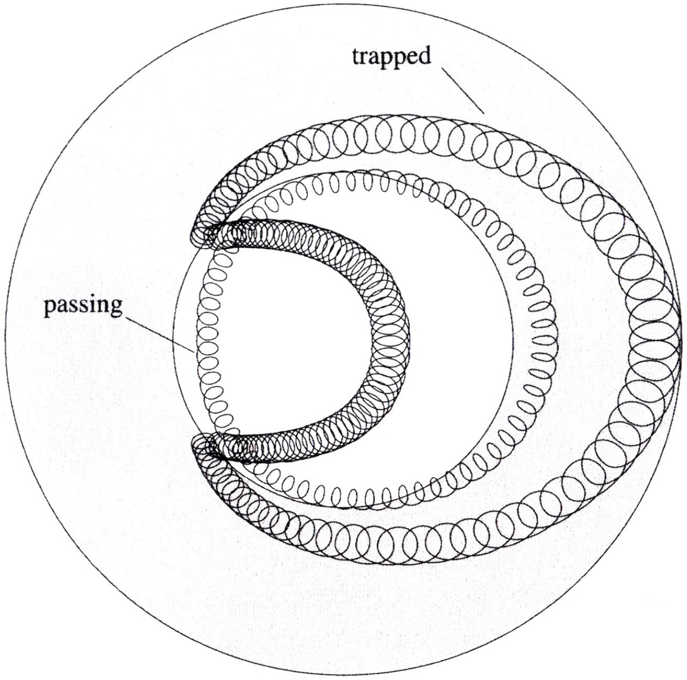
Banana regime transport I

- Banana regime transport is followed in plasmas with low collisionality. There, trapping of particles in the form of bananas is possible.
- According to the foregoing, particles moving in inhomogeneous magnetic field can be via „mirror effect“ reflected. Due to the symmetry, such particle will be trapped in the weaker region of the tokamak magnetic field and its projection of the trajectory on the plane, going through the axis, creates a closed curve, banana.
- Banana regime requires that the bounce frequency ω_b will be higher than the collision frequency ν_{ei}.
- In the following picture we present two examples of bananas, yet uninfluenced by collisions and passing particles.

Banana regime transport II

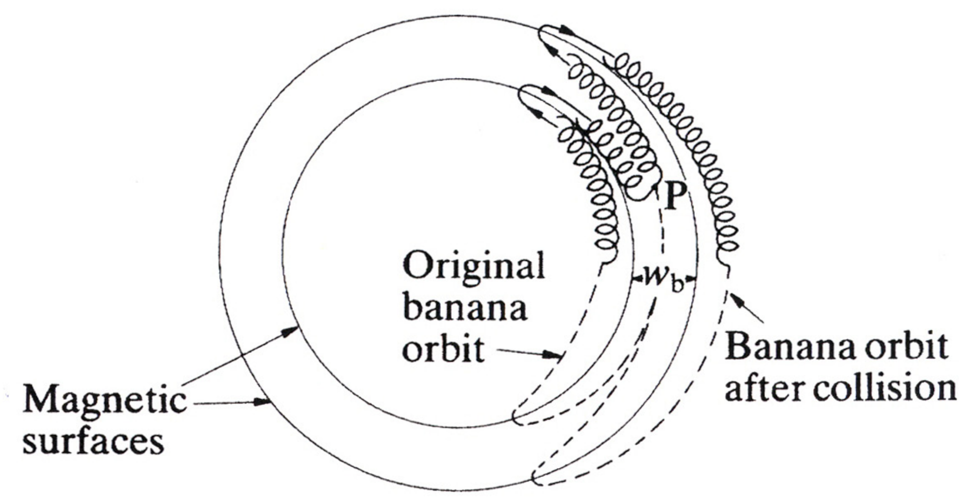






Banana regime transport III

The following picture shows schematically
the effect of the collision on the banana trajectory.



Banana regime transport IV

- To derive the corresponding diffusion coefficient of bananas diffusion, the following parameters must be declared:
  - w_{be} electron banana width
  - ν_{ef} effective collision frequency

- Electron banana width can be found from the solution of banana dynamics. From that,

$$w_{be} = \left(\frac{q}{\varepsilon^{1/2}} \right) \rho_e$$

- Here, ε is the inverse aspect ratio and ρ_e is electron Larmor radius.

Banana regime V

- Effective collision frequency for detrapping is $\nu_{ef} = \dfrac{\nu_{ei}}{\varepsilon}$
- Since the number of the trapped particles is $\approx \varepsilon^{1/2}$, the resulting diffusion coefficient *D* is

$$D \approx \frac{q^2}{\varepsilon^{3/2}} \nu_{ei} \rho_e^2$$

- This exceeds the classical diffusion coefficient by $q^2 \varepsilon^{-3/2}$ and the Pfirsch-Schluter coefficient by $\varepsilon^{-3/2}$

Plateau regime

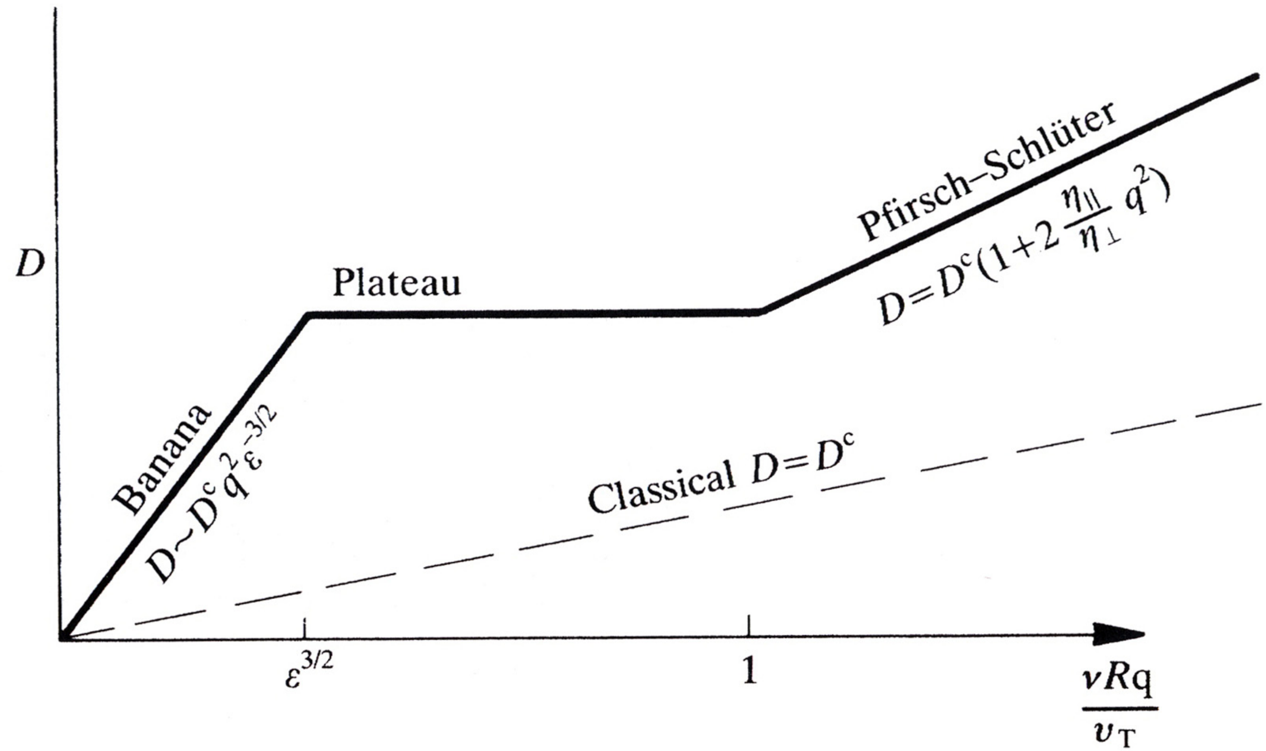
- Pfirsch-Schlüter and banana regime leave a gap in collision frequency

$$\frac{1}{q^{3/2}} v_{Te} \frac{1}{R} \rangle \ \nu_{ei} \ \rangle \nu_{be}$$

- In this region, the diffusion is found to be dominated by a class of slowly circulating particles.
- Then, a heuristic estimate of the diffusion coefficient *D* gives

$$D \approx \frac{\nu_{Te} q}{R} \rho^2$$

Variation of diffusion coefficient with collision frequency



Variation of diffusion coefficient with collision frequency; scaling of energy confinement time

- In all regimes, the diffusion coefficient is larger than the classical one. Nevertheless, even with this difference, experimentally detected diffusion exceeds our solution.
- The corresponding energy confinement time was found for banana regime as well as for Pfirsch-Schlüter in the form (*l* is the characteristic dimension)

$$\tau_E \approx \frac{T^{1/2} B_p^2}{n} l^2$$

Scaling of collisional systems and Goldston scaling

- The general behaviour can be illustrated by a scaling obtained by Goldston, which uses experimental data from a set of tokamaks (therefore not by an analytical approach)

$$\tau_E \approx \frac{B_p^2}{nT} l^{1.8}$$

- The difference between this scaling and the scaling for collisional plasma is remarkable.
- Whereas the scaling of diffusional systems depends on the temperature as $\sqrt{T}$, the foregoing scaling depends on the temperature as T^{-1}.
- This made the achievement of high temperature difficult.
- However, the H-modes gives chance for substantial improvement.

Bootstrap current

- There is yet one phenomenon, which has connection with banana regime – bootstrap current.
- Such a current exists independently on the convention tokamak current (e.g. inductive current in tokamaks). This current could provide a part of tokamak poloidal magnetic field – therefore the term „bootstrap current“
- Electron-ion momentum exchange, related to the higher transport in the banana regime, implies the existence of unidirectional current – bootstrap current.

Simple explanation of the bootstrap current I

For the inverse aspect ratio $\varepsilon = \frac{r}{R}$,

there is a fraction $\varepsilon^{1/2}$ of trapped particle in the banana

and have a typical parallel velocity $\varepsilon^{1/2} v_{Te}$

They execute a banana orbit of width $w_b \approx \varepsilon^{-1/2} q\rho$

Due to the presence of density gradient, these particle carry a current

$$j_t = -e\varepsilon^{1/2}\left(\varepsilon^{1/2} v_{Te}\right) w_b \frac{dn}{dr} \approx -q\frac{\varepsilon^{1/2}}{B} T \frac{dn}{dr}$$

Simple explanation of the bootstrap current II

- Both trapped ions and electrons carry such current. There is a transfer of their momentum to the passing particles – electron as well as ions. This transfer then adjust their velocities.
- The dominant current arises from the difference in velocities between the passing ions and passing electrons. This difference then results in the bootstrapped current.
- Using this procedure, we obtain the expression for the amount of this current (parallel to the magnetic field):

$$j_b = -\frac{\varepsilon^{1/2}}{B_\theta} T \frac{dn}{dr}$$

Conclusion I

- The short review on collisional diffusion in tokamak plasma represents a large theoretical effort, devoted to this problem. The diffusion was solved, using simple models, mainly analytically.
- The comparison with many experiments shows, nevertheless, discrepancies. The measured transport is always faster.
- It is now generally accepted that this discrepancies are caused by several types of instabilities, experimentally detected.
- Fortunately, the discovering of H-modes (Wagner F. et al., Physics Review Letters 49 1408 (1982)) shows a possibility of sufficient improving of the confinement, and, simultaneously, to construct a test thermonuclear tokamak reactor with acceptable dimensions and costs.

Conclusion II

- The new generation of transport properties is closely related to the experimental effort, mainly leaving the analytical approaches and using broadly numerical simulations.
- Is seems now that the main impetus for the discussion of transport properties is coming via experiments. This will be more broadly discussed in following presentation of Dr. Pánek.
- This leadership of experimental activity is understandable. The processes in tokamak plasma are extremely complex. To establish a unified theory of confinement of plasma in tokamaks still needs a lot of effort.
- Perhaps the most important part of this will consist in understanding of mechanisms of transport barriers.

Conclusions III

It seems that this status can be – *cum grano salis* – characterized by one well known sentence:

„Grau, teuer Freund, ist alle Theorie. Und grün des Lebens goldner Baum“

Mephistopheles und Schüler, J.W. Goethe: Faust I

Conclusions IV

Monographies used (and recommended for further study):

- Wesson J.: *Tokamaks*, International Series of Monographs on Physics 118, Oxford Science Publications, Clarendon Press Oxford 2004.
- Miyamoto K.: *Plasma Physics and Controlled Fusion*, Springer Series on Atomic,Optical and Plasma Physics, Springer 2005.
- Chen F.F.: *Introduction to Plasma Physics and Controlled Fusion*, Vol. 1: Plasma Physics, Springer 2006.
- Bellan P.M.: *Fundamentals of Plasma Physics*, Cambridge University Press, 2006.
- Boyd T.J.M., Sanderson J.J.: *The Physics of Plasmas*, Cambridge University Press, 2003.